In a completely analogous manner we can find a tight upper bound $\bar{p}$ for p by solving a minimization problem:

$$
(13.5)\qquad \begin{array}{ll} \text{minimize} & \bar{p} = x_0 + x_1 s_0 + \sum_{j=2}^{n} x_j p_j \\ \text{subject to} & x_0 + x_1 s_1(i) + \sum_{j=2}^{n} x_j h_j(s_1(i)) \geq g(s_1(i)), \quad i = 1, \ldots, m. \end{array}
$$

The dual problem associated with (13.4) is

$$
\begin{array}{ll} \text{minimize} & \sum_i g(s_1(i)) y_i \\ \text{subject to} & \sum_i y_i = 1, \\ & \sum_i s_1(i) y_i = s_0, \\ & \sum_i h_j(s_1(i)) y_i = p_j, \quad j = 2, \ldots, n \\ & y_i \geq 0, \quad i = 1, \ldots, m. \end{array}
$$

Note that the first and last constraints tell us that the y_i's are a system of probabilities. Given this interpretation of the y_i's as probabilities, the expression

$$
\sum_i s_1(i) y_i
$$

is just an expected value of the random variable S_1 computed using these probabilities. So, the constraint $\sum_i s_1(i) y_i = s_0$ means that the expected stock price at the end of the time period must match the current stock price, when computed with the y_i probabilities. For this reason, we call these probabilities *risk neutral*. Similarly, the constraints $\sum_i h_j(s_1(i)) y_i = p_j$, $j = 2, \ldots, n$, tell us that each of the options must also be priced in such a way that the expected future price matches the current market price.

Exercises

13.1 Find every portfolio on the efficient frontier using the most recent 6 months of data for the Bond (SHY), Materials (XLB), Energy (XLE), and Finacial (XLF) sectors as shown in Table 13.1 (that is, using the upper left 6×4 subblock of data).

13.2 On Planet Claire, markets are highly volatile. Here's some recent historical data:

Year-Month	Hair Products	Cosmetics	Cash
2007-04	1.0	2.0	1.0
2007-03	2.0	2.0	1.0
2007-02	2.0	0.5	1.0
2007-01	0.5	2.0	1.0

Find every portfolio on Planet Claire's efficient frontier.

13.3 What is the dual of (13.5)?

Notes

The portfolio selection problem originates with Markowitz (1959). He won the 1990 Nobel prize in Economics for this work. In its original formulation, risk is modeled by the variance of the portfolio's value rather than the absolute deviation from the mean considered here. We will discuss the quadratic formulation later in Chapter 24.

The MAD risk measure we have considered in this chapter has many nice properties the most important of which is that it produces portfolios that are guaranteed not to be stochastically dominated (to second order) by other portfolios. Many risk measures fail to possess this important property. See Ruszczyński & Vanderbei (2003) for details.