Exercises

19.1 *Sherman–Morrison–Woodbury Formula*. Assuming that all the inverses below exist, show that the following identity is true:

$$(E^{-1} + ADA^T)^{-1} = E - EA(A^TEA + D^{-1})^{-1}A^TE.$$

Use this identity to verify directly the equivalence of the expressions given for Δx in (19.11) and (19.12).

19.2 Assuming that all the inverses exist, show that the following identity holds:

$$I - (E + ADA^T)^{-1}ADA^T = (E + ADA^T)^{-1}E.$$

19.3 Show that

$$\Delta w = \Delta w_{\text{OPT}} + \mu \Delta w_{\text{CTR}} + \Delta w_{\text{FEAS}},$$

where

$$\begin{aligned}
\Delta w_{\text{OPT}} &= -A\left(D^2 - D^2A^T(E^{-2} + AD^2A^T)^{-1}AD^2\right)c, \\
\Delta w_{\text{CTR}} &= -A\left(D^2 - D^2A^T(E^{-2} + AD^2A^T)^{-1}AD^2\right)X^{-1}e \\
&\qquad + AD^2A^T(E^{-2} + AD^2A^T)^{-1}E^{-2}W^{-1}e,
\end{aligned}$$

and

$$\Delta w_{\text{FEAS}} = \rho - AD^2A^T(E^{-2} + AD^2A^T)^{-1}\rho.$$

Notes

The KKT system for general inequality constrained optimization problems was derived by Kuhn & Tucker (1951). It was later discovered that W. Karush had proven the same result in his 1939 master's thesis at the University of Chicago (Karush 1939). John (1948) was also an early contributor to inequality-constrained optimization. Kuhn's survey paper (Kuhn 1976) gives a historical account of the development of the subject.