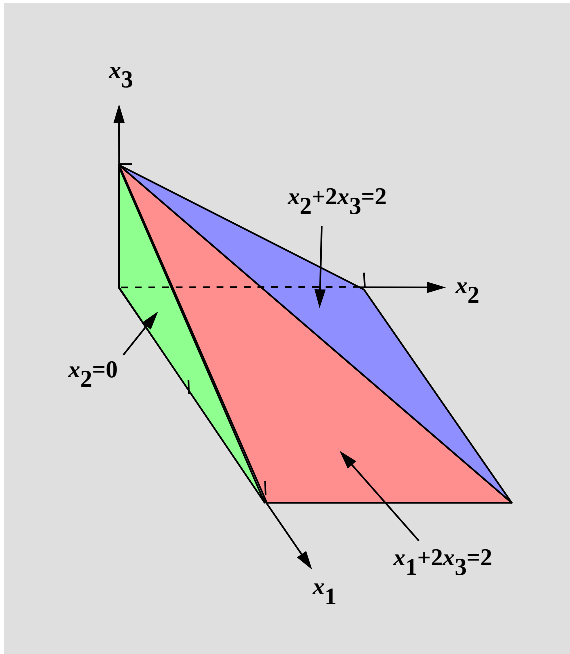

FIGURE 3.2. The set of feasible solutions for the (degenerate) problem given by (3.7).

Exercises

3.1 Solve the following linear program using the perturbation method to resolve degeneracy:

$$\begin{array}{ll} \text{maximize} & 10x_1 - 57x_2 - 9x_3 - 24x_4 \\ \text{subject to} & 0.5x_1 - 5.5x_2 - 2.5x_3 + 9x_4 \leq 0 \\ & 0.5x_1 - 1.5x_2 - 0.5x_3 + x_4 \leq 0 \\ & x_1 \leq 1 \\ & x_1,\ x_2,\ x_3,\ x_4 \geq 0. \end{array}$$

Note: The simple pivot tool with the *Lexicographic* labels can be used to check your arithmetic:

www.princeton.edu/∼rvdb/JAVA/pivot/simple.html

3.2 Solve the following linear program using Bland's rule to resolve degeneracy:

$$
\begin{array}{ll}
\text{maximize} & 10x_1 - \ 57x_2 - \ \ 9x_3 - 24x_4 \\
\text{subject to} & 0.5x_1 - 5.5x_2 - 2.5x_3 + \ 9x_4 \le 0 \\
& 0.5x_1 - 1.5x_2 - 0.5x_3 + \ \ x_4 \le 0 \\
& x_1 \qquad\qquad\qquad\qquad\qquad\quad \le 1 \\
& \qquad\qquad x_1,\ x_2,\ x_3,\ x_4 \ge 0.
\end{array}
$$

3.3 Using today's date (MMYY) for the seed value, solve 10 possibly degenerate problems using the online pivot tool:

www.princeton.edu/∼rvdb/JAVA/pivot/lexico.html

3.4 Consider the linear programming problems whose right-hand sides are identically zero:

$$
\begin{array}{lll}
\text{maximize} & \displaystyle\sum_{j=1}^{n} c_j x_j & \\
\text{subject to} & \displaystyle\sum_{j=1}^{n} a_{ij} x_j \le 0 & i = 1, 2, \ldots, m \\
& x_j \ge 0 & j = 1, 2, \ldots, n.
\end{array}
$$

Show that either $x_j = 0$ for all j is optimal or else the problem is unbounded.

3.5 Consider the following linear program:

$$
\begin{array}{ll}
\text{maximize} & x_1 + 3x_2 \\
\text{subject to} & -2x_1 \qquad \le -5 \\
& \qquad\quad x_1 \ge \ \ 0\,.
\end{array}
$$

Show that this problem has feasible solutions but no vertex solutions. How does this reconcile with the fundamental theorem of linear programming (Theorem 3.4)?

3.6 Suppose that a linear programming problem has the following property: its initial dictionary is not degenerate and, when solved by the simplex method, there is never a tie for the choice of leaving variable.

(a) Can such a problem have degenerate dictionaries? Explain.
(b) Can such a problem cycle? Explain.

3.7 Consider the following dictionary:

$$\begin{array}{rl} \zeta = & 5 + 2x_2 - 2x_3 + 3x_5 \\ \hline x_6 = & 4 - 2x_2 - \ \ x_3 + \ \ x_5 \\ x_4 = & 2 - \ \ x_2 + \ \ x_3 - \ \ x_5 \\ x_1 = & 6 - 2x_2 - 2x_3 - 3x_5. \end{array}$$

(a) List all pairs (x_r, x_s) such that x_r could be the entering variable and x_s could be the leaving variable.

(b) List all such pairs if the largest-coefficient rule for choosing the entering variable is used.

(c) List all such pairs if Bland's rule for choosing the entering and leaving variables is used.

Notes

The first example of cycling was given by Hoffman (1953). The fact that any linear programming problem that cycles must have at least six variables and three constraints was proved by Marshall & Suurballe (1969).

Early proofs of the fundamental theorem of linear programming (Theorem 3.4) were constructive, relying, as in our development, on the existence of a variant of the simplex method that works even in the presense of degeneracy. Hence, finding such variants occupied the attention of early researchers in linear programming. The *perturbation method* was first suggested by A. Orden and developed independently by Charnes (1952). The essentially equivalent *lexicographic method* first appeared in Dantzig et al. (1955). Theorem 3.3 was proved by Bland (1977).

For an extensive treatment of degeneracy issues see Gal (1993).