Exercises

In solving the following problems, the advanced pivot tool can be used to check your arithmetic:

www.princeton.edu/∼rvdb/JAVA/pivot/advanced.html

7.1 The final dictionary for

$$\begin{array}{ll} \text{maximize} & x_1 + 2x_2 + x_3 + x_4 \\ \text{subject to} & 2x_1 + x_2 + 5x_3 + x_4 \leq 8 \\ & 2x_1 + 2x_2 + 4x_4 \leq 12 \\ & 3x_1 + x_2 + 2x_3 \leq 18 \\ & x_1,\ x_2,\ x_3,\ x_4 \geq 0 \end{array}$$

is

$$\begin{array}{rl} \zeta = & 12.4 - 1.2x_1 - 0.2x_5 - 0.9x_6 - 2.8x_4 \\ \hline x_2 = & 6 - x_1 - 0.5x_6 - 2x_4 \\ x_3 = & 0.4 - 0.2x_1 - 0.2x_5 + 0.1x_6 + 0.2x_4 \\ x_7 = & 11.2 - 1.6x_1 + 0.4x_5 + 0.3x_6 + 1.6x_4. \end{array}$$

(the last three variables are the slack variables).

(a) What will be an optimal solution to the problem if the objective function is changed to

$$3x_1 + 2x_2 + x_3 + x_4?$$

(b) What will be an optimal solution to the problem if the objective function is changed to

$$x_1 + 2x_2 + 0.5x_3 + x_4?$$

(c) What will be an optimal solution to the problem if the second constraint's right-hand side is changed to 26?

7.2 For each of the objective coefficients in the problem in Exercise 7.1, find the range of values for which the final dictionary will remain optimal.

7.3 Consider the following dictionary which arises in solving a problem using the self-dual simplex method:

$$\begin{array}{rl} \zeta = & -3 - (-1 + 2\mu)x_1 - (3 - \mu)x_3 \\ \hline x_2 = & -1 + \mu + x_1 - x_3 \\ x_4 = & -4 + 3\mu + 3x_1 - 2x_3 \\ x_5 = & 2 + x_1 + x_3\,. \end{array}$$

(a) For which values of μ is the current dictionary optimal?

(b) For the next pivot in the self-dual simplex method, identify the entering and the leaving variable.

7.4 Solve the linear program given in Exercise 2.3 using the self-dual simplex method. *Hint: It is easier to use dictionary notation than matrix notation.*

7.5 Solve the linear program given in Exercise 2.4 using the self-dual simplex method. *Hint: It is easier to use dictionary notation than matrix notation.*

7.6 Solve the linear program given in Exercise 2.6 using the self-dual simplex method. *Hint: It is easier to use dictionary notation than matrix notation.*

7.7 Using today's date (MMYY) for the seed value, solve 10 problems using the self-dual simplex method:

www.princeton.edu/∼rvdb/JAVA/pivot/pd1phase.html

7.8 Use the self-dual simplex method to solve the following problem:

$$\begin{array}{ll} \text{maximize} & 3x_1 - x_2 \\ \text{subject to} & \begin{aligned} x_1 - x_2 &\leq 1 \\ -x_1 + x_2 &\leq -4 \\ x_1,\ x_2 &\geq 0. \end{aligned} \end{array}$$

7.9 Let P_μ denote the perturbed primal problem (with perturbation μ). Show that if P_μ is infeasible, then $P_{\mu'}$ is infeasible for every $\mu' \leq \mu$. State and prove an analogous result for the perturbed dual problem.

7.10 Using the notation of Figure 7.1 state precise conditions for detecting infeasibility and/or unboundedness in the self-dual simplex method.

7.11 Consider the following one parameter family of linear programming problems (parametrized by μ):

$$\begin{array}{lrcrcrcrcrcl} \max & (4-4\mu)x_0 & - & 2x_1 & - & 2x_2 & - & 2x_3 & - & 2x_4 & & \\ \text{s.t.} & x_0 & - & x_1 & & & & & & & \leq & 1 \\ & x_0 & & & - & x_2 & & & & & \leq & 2 \\ & x_0 & & & & & - & x_3 & & & \leq & 4 \\ & x_0 & & & & & & & - & x_4 & \leq & 8 \\ & & & & & \multicolumn{5}{r}{x_0,\ x_1,\ x_2,\ x_3,\ x_4} & \geq & 0. \end{array}$$

Starting from $\mu = \infty$, use the parametric simplex method to decrease μ as far as possible. Don't stop at $\mu = 0$. If you cannot get to $\mu = -\infty$, explain why. *Hint: the pivots are straight forward and, after the first couple, a clear* *pattern should emerge which will make the subsequent pivots easy.* Clearly indicate the range of μ values for which each dictionary is optimal.

Notes

Parametric analysis has its roots in Gass & Saaty (1955). G.B. Dantzig's classic book (Dantzig 1963) describes the self-dual simplex method under the name of the *self-dual parametric simplex method*. It is a special case of "Lemke's algorithm" for the linear complementarity problem (Lemke 1965) (see Exercise 18.7). Smale (1983) and Borgwardt (1982) were first to realize that the parametric self-dual simplex method is amenable to probabilistic analysis. For a more recent discussion of homotopy methods and the parametric self-dual simplex method, see Nazareth (1986) and Nazareth (1987).