Newton & Kepler

Freshman Seminar 131

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Orbital Mechanics

Here's a link to my Sky & Telescope article: Measuring the Astronomical Unit

Isaac Newton:

Newton's Second Law of Motion:

$$F = ma$$

Newton's Law of Gravity:

$$\|F\| = G\frac{mM}{r^2}$$

Johannes Kepler:

Orbital Period (Kepler's Third Law):

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

Kepler's Third Law

Kepler's Law

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

If orbital radius r is measured in Sun/Earth distance (aka *astronomical units*) and orbital period is measured in years, then

$$1 = 2\pi\sqrt{\frac{1^3}{GM}}$$

Hence, Orbital Period (Kepler's Third Law):

$$T = \sqrt{r^3}$$

We can also write it like this:

$$r = T^{2/3}$$

Asteroid Prudentia

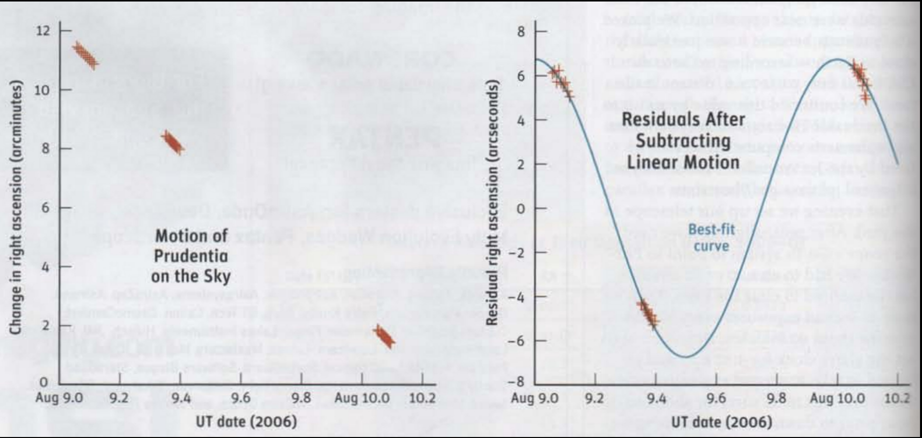
to be a nearly-24-hour sine curve. Shown in the diagram is the curve of this type that best fits our data.

Our observing site was at latitude 40° 27′ north. Simple trigonometry tells us that the baseline for this latitude is 9,693 km (the diameter of Earth is 12,738 km, which we multiply by the cosine of our latitude to get the baseline). From the diagram below, it's clear that the peak-to-peak RA oscillation (θ) was about 13 arcseconds. To determine θ as accurately as we could, we did a regression analysis (using a quadratic rather than a simple linear equation for the background motion), which gave a peak-to-peak value of 14.47 arcseconds.

Given the known baseline (*b*) in kilometers and the angle, we could compute the distance (*d*) in kilometers to Prudentia using the formula $d = b/2\sin(\theta/2)$, which gives 138,200,000 km. Therefore 1 a.u. equals 138,200,000 km divided by 0.9309 a.u. (the ephemeris distance that we looked up), or 148,500,000 km. That's only 0.7% below the correct value!

In just 26 hours and armed only with hobby equipment, we measured the a.u. almost as well as David Gill did — with a small fraction of his skill and effort. The result could surely be improved by conducting many observations over many nights, and from a site closer to the equator (giving a longer baseline), and in better seeing. But it's amazing to realize how easy it is these days to measure a fundamental quantity that many ancient and medieval astronomers would have gladly given their lives to learn.

Asteroid Prudentia



Click on the diagram or here to see the Sky & Telescope article.