(1) Consider the following dictionary which arises in solving a problem using the primal–dual one–phase simplex method:

$$
\zeta = -3 - (-1 + 2\mu)x_1 - (3 - \mu)x_3
$$

$$
x_2 = -1 + \mu + x_1 - x_3
$$

$$
x_4 = -4 + 3\mu + 3x_1 - 2x_3
$$

$$
x_5 = 2 + x_1 + x_3
$$

(a) For which values of $\mu$ is the current dictionary optimal?

(b) For the next pivot in the primal–dual one–phase simplex method, identify the entering and the leaving variable.

(2) Consider a linear programming problem that has an optimal dictionary in which exactly $k$ of the original slack variables are nonbasic. Show that, by ignoring feasibility preservation of intermediate dictionaries, this dictionary can be arrived at in exactly $k$ pivots. Hint: Don’t forget to allow for the fact that some pivot elements might be zero.