Optimal Shaped Pupil Coronagraphs for Extrasolar Planet Finding

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August 26, 2002
Objective

Create a system Point Spread Function (PSF) with the needed contrast allowing planet discovery at the smallest inner working distance (IWD) in the shortest integration time.

Outline

- Point Spread Function for Equal-Area Apertures
- Performance Metrics
- Apodized Pupils Comparisons
- Optimal Shaped Pupil Designs

Note: We assume systematic error is separately reduced to below background.
Background

Irradiance $I(\xi, \zeta)$ is an intensity $I_o$ times the point spread function $P(\xi, \zeta)$:

$$I(\xi, \zeta) = I_o P(\xi, \zeta)$$

The intensity $I_o$ depends only on the source strength and the aperture area $A = a^2$.

The point spread function (PSF) for a $a\sqrt{r} \times a/\sqrt{r}$ rectangular design space is given by

$$P(\xi, \zeta) = \left| \int_{-\frac{1}{2\sqrt{r}}}^{\frac{1}{2\sqrt{r}}} \int_{-\frac{\sqrt{r}}{2}}^{\frac{\sqrt{r}}{2}} A(y, z) e^{-2\pi i (\xi y + \zeta z)} dy dz \right|^2,$$

where the function $A()$ denotes the apodization function.

We distinguish purely open/closed pupils from smooth apodizations:

$$A \in \{0, 1\} \implies \text{pupil mask}$$

$$A \in [0, 1] \implies \text{smooth apodization}.$$
Optimization Criteria

We compare and optimize different designs with equal pre-apodized aperture area.

Comparisons are made on four performance metrics:

- Contrast
- Integration Time
- Inner Working Distance
- Discovery Space
Brown and Burrows (Icarus 1990) introduced the contrast quotient:

\[
Q = \frac{I_{\text{planet}} P(0, 0)}{I_{\text{star}} P(\xi_{\text{planet}}, \zeta_{\text{planet}}) + I_{\text{scatter}}}, = \frac{I_{\text{planet}} P(0, 0)}{I_{\text{bkgrd}}}
\]

where \( I_{\text{planet}} \) denotes the planet’s irradiance, \( I_{\text{star}} \) denotes the star’s irradiance, and \((\xi_{\text{planet}}, \zeta_{\text{planet}})\) denotes the image-plane coordinates of the planet relative to an on-axis parent star.

It is estimated that \( I_{\text{planet}}/I_{\text{star}} \) might be as small as \(10^{-10}\). Hence, to have a unit contrast ratio, \( Q = 1 \), we require

\[
P(\xi_{\text{planet}}, \zeta_{\text{planet}}) \leq 10^{-10} P(0, 0).
\]
Integration Time

Detection in Known Background

Assume Poisson arrival statistics for planet signal and background:

\[
\frac{S}{N} = \frac{I_{\text{planet}} t \Delta P}{\sqrt{I_{\text{bkgrd}} t \Delta S}} \implies t_1 = \frac{(S/N)^2 P(0,0) \Delta S}{I_{\text{planet}} Q \Delta P^2},
\]

where \( \Delta P \) denotes the area under the PSF in a neighborhood \( \Delta S \) of \((0,0)\).

Photometry in Unknown Background (Characterization)

\[
t_2 = \frac{(S/N)^2 P(0)}{I_{\text{planet}}} \left\{ \frac{\Delta S^2 \Delta^3 P - (\Delta P)^3}{P(0,0) \left[ \Delta S \Delta^2 P - (\Delta P)^2 \right]} + \frac{\Delta S}{Q} \right\}
\]
Inner Working Distance (IWD)

The smallest angular separation from the star for which the PSF contrast reaches the required value of $10^{-10}$.

Discovery Space

The azimuthal region of the image plane within which discovery is possible; that is, where $10^{-10}$ contrast is achieved.
Apodized Pupils


Left The PSF for an ASA with a sonine apodization ($\nu = 5$) plotted on a logarithmic scale with black areas $10^{-10}$ below brightest. Right Cross section of ASA on diagonal showing inner working distance of $5\lambda/a$. Single-exposure integration time of $t_1 = 25.5$. 
On-diagonal, IWD ≈ 5λ/a. Single-exp. integration time of t₁ = 9.5.
Generalized Prolate Spheroidal (GPS) Apodization

Note: Circular Telescope

IWD = 3.5λ/a. Single-exposure integration time of $t_1 = 17$. 

Cross Section of Generalized Prolate Spheroidal Apodization

Cross Section Angle (λ/a)
### Integration Times for Apodizations

<table>
<thead>
<tr>
<th>Type</th>
<th>$\frac{t_1I_p}{(S/N)^2}$</th>
<th>$\frac{t_2I_p}{(S/N)^2}$</th>
<th>$IWD$</th>
<th>Disc. Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sonine, $\nu = 5$</td>
<td>25.5</td>
<td>48</td>
<td>5 (diag), 13 (axis)</td>
<td>&lt; 1/2</td>
</tr>
<tr>
<td>Sonine, $\nu = 4$</td>
<td>19</td>
<td>35</td>
<td>7 (diag), 15 (axis)</td>
<td>&lt; 1/2</td>
</tr>
<tr>
<td>1-D Prolate</td>
<td>9.5</td>
<td>16.5</td>
<td>5 (diag), 4 (axis)</td>
<td>1/2</td>
</tr>
<tr>
<td>Gen. Prolate</td>
<td>17</td>
<td>34</td>
<td>3.5</td>
<td>full</td>
</tr>
</tbody>
</table>

Integration time comparisons for four different pupil apodizations in equal area apertures. Integration times are for a single exposure and have been normalized by planet irradiance and signal-to-noise ratio. The integration time $t_1$ assumes a known background level and is for planet discovery only. The integration time $t_2$ simultaneously estimates planet and background irradiance by using a region of the image plane slightly larger than the full width of the main lobe.
Prolate Spheroidal with White Apod. Error of $6 \times 10^{-12} \text{ Hz}^{-2}$

Four orders of magnitude degradation.
Shaped Pupil Masks

Prolate Spheroidal Mask

\[ \text{IWD} = 4\lambda/a. \] Single-exposure integration time of 4.6.
6-Pupil Rectangular

IWD = 4.2\lambda/a. Single-exposure integration time of \( t_1 = 15 \).
6-Pupil Elliptical

$IWD = 2.8\lambda/a$. Single-exposure integration time of $t_1 = 6.5$. 
Larger OWD than the previous mask. IWD = 4.5λ/a. Single-exposure integration time of $t_1 = 4.8$. 

Another 6-Pupil Elliptical
8 Pupil Circular Eclipse-Class (Black = $10^{-8}$)

IWD = 3.5λ/a. Single-exposure integration time of $t_1 = 7.9$. 
Azimuthally Symmetric Mask

IWD = 3.5λ/a. Single-exposure integration time of \( t_1 = 18 \).
Integration Times for Masks

<table>
<thead>
<tr>
<th>Type</th>
<th>$\frac{t_1 I_p}{(S/N)^2}$</th>
<th>$\frac{t_2 I_p}{(S/N)^2}$</th>
<th>IWD</th>
<th>Discovery Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>single prolate</td>
<td>4.6</td>
<td>8</td>
<td>4</td>
<td>$\ll 1/2$</td>
</tr>
<tr>
<td>6 pupil rectangle</td>
<td>15</td>
<td>21</td>
<td>4.2</td>
<td>1/2</td>
</tr>
<tr>
<td>6 pupil ellipse small OWD</td>
<td>6.5</td>
<td>9.5</td>
<td>2.8</td>
<td>1/4</td>
</tr>
<tr>
<td>6 pupil ellipse large OWD</td>
<td>4.8</td>
<td>7</td>
<td>4.5</td>
<td>1/4</td>
</tr>
<tr>
<td>8 pupil Eclipse</td>
<td>7.9</td>
<td>11</td>
<td>3.5</td>
<td>1/2</td>
</tr>
<tr>
<td>Concentric Rings</td>
<td>18</td>
<td>25</td>
<td>3.5</td>
<td>full</td>
</tr>
</tbody>
</table>

Integration time comparisons for three different shaped pupil apodizations in equal area apertures. Integration times are for a single exposure and have been normalized by planet irradiance and signal-to-noise ratio. The integration time $t_1$ assumes a known background level and is for planet discovery only. The integration time $t_2$ simultaneously estimates planet and background irradiance by using a region of the image plane slightly larger than the full width of the main lobe.
Conclusions

SHAPED PUPIL MASKS RULE! Here’s why:

- Better integration time, inner working distance, discovery space.
- Easier to manufacture.
- Less susceptibility to space environment.
- Flexible design.
  - On or off axis.
  - Discovery vs. characterization.
- Advantages of tailoring the PSF.
  - High PSF contrast implies high speckle contrast.
  - Insensitivity to jitter and stellar size.
- The best shaped pupils are as good as or better than the best graded apodizations.