Diffraction Analysis of Apodized Pupil Mapping Systems

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1. Summary

• Pure pupil mapping (aka PIAA) is similar to apodization.

• Advantages:
  – Ideally 100% throughput (vs. about 10% for apodization).
  – Very small inner working angle (vs. about $4\lambda/D$ for apodization).

• Disadvantages:
  – Doesn’t achieve $10^{-10}$ contrast.
  – Not an imaging system.
  – Hard to manufacture?

• Apodized Pupil Mapping (aka hybrid PIAA) resolves first disadvantage.

• Star occulter with reversed system resolves second disadvantage.

• Sensitivity to Zernike perturbations are shown.
Pupil Mapping via Ray Optics

\[ \tilde{h}(\tilde{r}) \]

\[ \tilde{r} \]

\[ h(r) \]

\[ R(\tilde{r}) \]

\[ S(R(\tilde{r}), \tilde{r}) \]

\[ \tilde{R}(r) \]

\[ \tilde{h}(\tilde{r}) \]
High-Contrast Amplitude Profiles
Diffraction Analysis: Pupil Mapping

Yikes!! Contrast lost.
Contrast regained. Slightly chromatic.
Occulter and Wavefront Reconstruction
On-Axis PSF at First and Second Focus

![Graph showing On-Axis PSF at First and Second Focus](image)
Off-axis Planet Images

<table>
<thead>
<tr>
<th>tilt = 0.0009 λ/D</th>
<th>0.0144 λ/D</th>
<th>0.1155 λ/D</th>
<th>1 λ/D</th>
<th>2 λ/D</th>
<th>3 λ/D</th>
<th>4 λ/D</th>
<th>5 λ/D</th>
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<td>1st FOCUS</td>
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<td>SHAPED PUPIL</td>
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Cross-Sectional Plots

Log$_{10}$ of Intensity relative to response with no obstruction

working angle in units of $\lambda/D$

Tilt of $4\lambda/D$
$2\lambda/D$
$1\lambda/D$
Off-Axis Source Attenuation

- Pupil mapping
- Concentric ring shaped pupil

Total power throughput vs. working angle in units of $\lambda/D$. The graph shows the performance of two different types of pupil mapping, with the blue line representing pupil mapping and the red line representing a concentric ring shaped pupil.
Zernike Sensitivities: Apodized Pupil Mapping

1/100th wave rms
Zernike Sensitivities: Concentric Ring Mask

(0,0) (1,1) (2,0)
(2,2) (3,1) (3,3)
(4,0) (4,2) (4,4)
Radial Profiles from Previous Two Slides
Contrast Degradation vs. RMS Error

(1,1)

(2,0)

(2,2)

(3,1)

(3,3)

(4,0)

(4,2)

(4,4)

Contrast

Rms of aberration in units of wave

Concentric rings, 4 \(\lambda/D\)
Concentric rings, 8 \(\lambda/D\)
Pupil mapping, 2 \(\lambda/D\)
Pupil mapping, 4 \(\lambda/D\)
Pupil mapping, 8 \(\lambda/D\)