Fresnel Analysis of Shaped Pupil Systems for Planet Finding

Robert J. Vanderbei
R. Belikov
N. J. Kasdin

SPIE San Diego
Aug 4, 2005

Princeton University
http://www.princeton.edu/~rvdb
Contents

1  Shaped Pupils For High-Contrast  3
2  Ideal PSF  4
3  Princeton Testbed  5
4  PSF Measured in the Lab  7
5  Fresnel Propagation  9
6  Intensity/Phase Profiles in Front of Pupil Mask  10
7  Intensity Profile at Pupil Mask  11
8  Fresnel PSF  12
9  Removing the 1” Aperture Stop  14
10 Conclusions  16
Shaped Pupils For High-Contrast

Elliptical Shaped Pupil Mask
Black is $10^{-10}$.
If the entering wavefront has no amplitude/phase error, then this is what you get.
Let's go planet hunting!!!
Princeton Testbed
Idealized Upstream Layout

5-micron Single mode fiber
1-inch aperture
6-inch off-axis Parabolic mirror
4-inch fold mirror
1-inch Elliptical Shaped pupil
Focusing element
Image plane

16" 44" 47" 23" 30"
PSF Measured in the Lab
Error is mostly phase/amplitude errors (1/20-th wave mirrors). Need DMs to correct these errors. Don’t have them yet. While waiting, we ask...

How much is caused by fundamental physics—i.e., *diffraction*?
5. Fresnel Propagation

Fresnel propagation of an electric field \( E_0 \) a distance \( z \) after a lens/mirror of focal length \( f \):

\[
E_1(\tilde{x}, \tilde{y}) = \frac{i}{\lambda z} e^{\frac{\pi i}{\lambda z} (\tilde{x}^2 + \tilde{y}^2)} \iint e^{\frac{\pi i}{\lambda f} (x^2 + y^2)} e^{-\frac{2\pi i}{\lambda z} (x\tilde{x} + y\tilde{y})} E_0(x, y) dy dx.
\]

If the lens/mirror is absent, then \( f = \infty \) and the middle exponential term in the integral disappears.
6. Intensity/Phase Profiles in Front of Pupil Mask

![Intensity Profile](image1)

![Phase Profile](image2)
7. Intensity Profile at Pupil Mask

Fresnel Intensity Profile

Fresnel Intensity Profile

-0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
Fresnel PSF
Removing the 1” Aperture Stop

![Graph of true Psf]
Conclusions

- Mirror errors limit contrast to about $10^{-6}$ at $5\lambda/D$.
- Diffraction effects limit contrast to about $10^{-8}$ for 1 inch apertures.
- Consider placing $8 \times 3.5$ meter shaped pupil at entrance pupil.
- Consider activating primary and secondary mirrors.
- All other designs might be too hard or even impossible.