Diffraction Analysis of 2-D Pupil Remapping for High-Contrast Imaging

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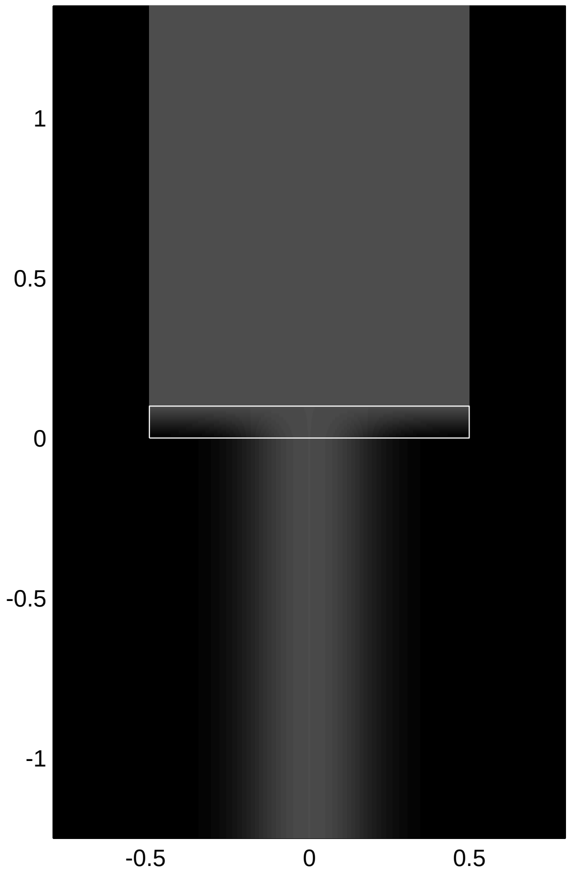
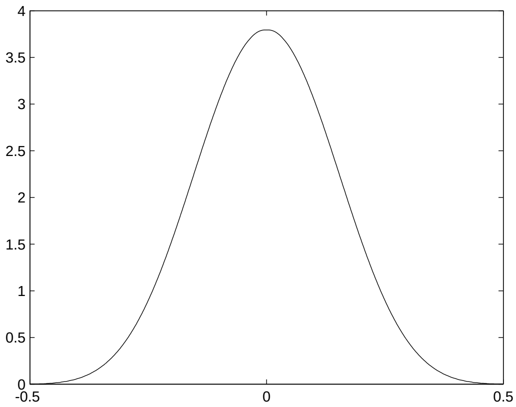
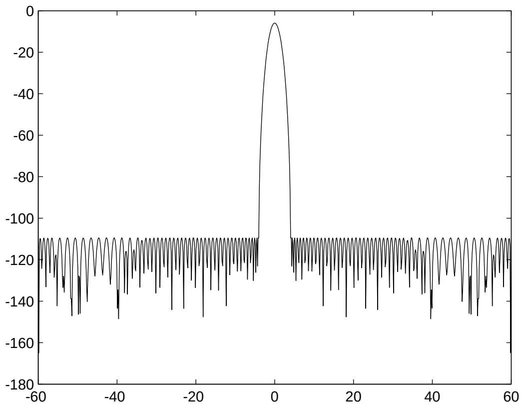
Apodization







$$E(\xi,\eta) = \frac{1}{\lambda f}\iint e^{2\pi i \frac{\tilde{x}\xi+\tilde{y}\eta}{\lambda f}} A(\sqrt{\tilde{x}^2+\tilde{y}^2})d\tilde{y}d\tilde{x}$$

$$E(\rho) = \frac{2\pi}{\lambda f}\int J_0\left(2\pi\frac{\tilde{r}\rho}{\lambda f}\right)A(\tilde{r})\tilde{r}d\tilde{r}.$$

$$\mathsf{Psf}(\rho) = |E(\rho)|^2$$

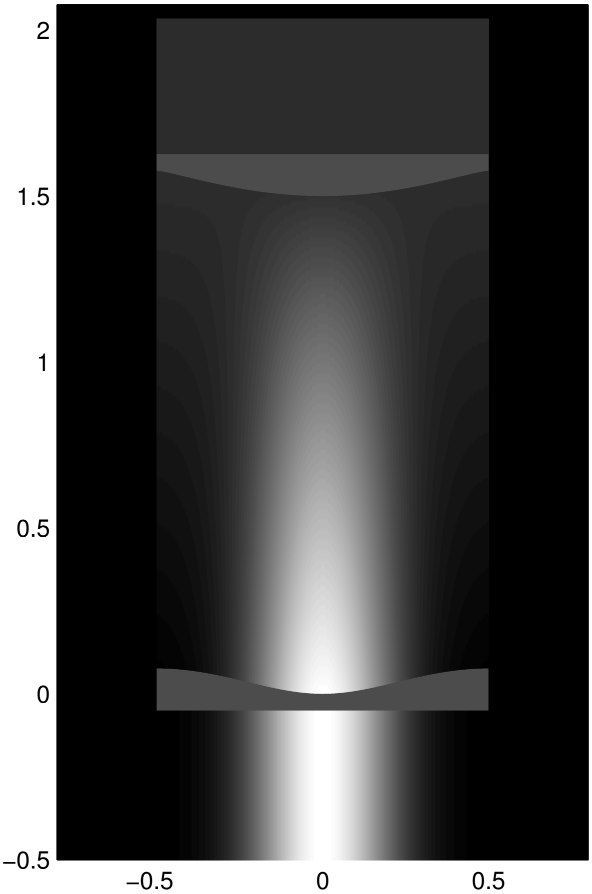
Pupil Mapping

Advantages:

- 100% throughput
- Implicit magnification... effectively iwa $\approx 1\lambda/D$.

Disadvantages:

- Diffraction effects limit achievable contrast to 10^{-5} for a pure pupil-mapping system.



Notations

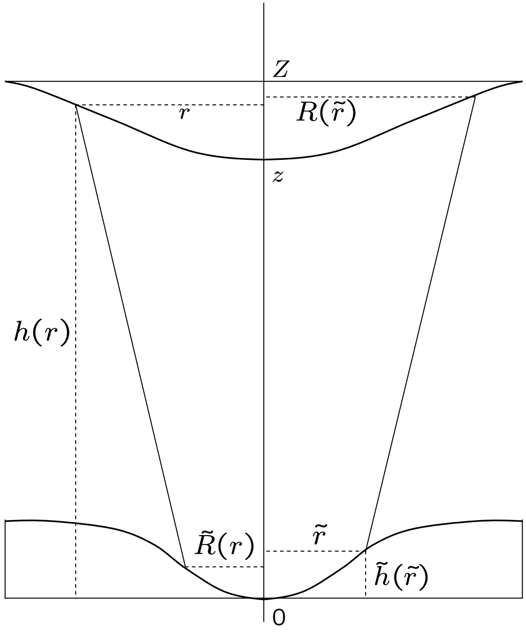
$$R(\tilde{r}) = \pm\sqrt{\int_0^{\tilde{r}} 2A^2(s)sds}.$$

$$\frac{\partial h}{\partial r}(r) = \frac{r - \tilde{R}(r)}{\sqrt{Q_0^2 + (n^2-1)(r-\tilde{R}(r))^2}}$$

$$\frac{\partial \tilde{h}}{\partial \tilde{r}}(\tilde{r}) = \frac{R(\tilde{r}) - \tilde{r}}{\sqrt{Q_0^2 + (n^2-1)(R(\tilde{r})-\tilde{r})^2}}$$

n is the refractive index and

$$\begin{aligned} Q_0 &= -(n-1)z \\ &= S(R(\tilde{r}),\tilde{r}) + n(\tilde{h}(\tilde{r}) - h(R(\tilde{r}))) \end{aligned}$$



Notes:

- the second expression for Q_0 is in fact independent of $\tilde{r}$.
- for mirrors, put $n = -1$.

Huygens Wavelets

Cartesian Coordinates

$$E_{\text{out}}(\tilde{x}, \tilde{y}) = \iint \frac{1}{\lambda Q(\tilde{x}, \tilde{y}, x, y)} e^{2\pi i Q(\tilde{x},\tilde{y},x,y)/\lambda} dy dx,$$

where Q denotes the optical path length:

$$Q(\tilde{x}, \tilde{y}, x, y) = \sqrt{(x-\tilde{x})^2 + (y-\tilde{y})^2 + (h(r) - \tilde{h}(\tilde{r}))^2} + n(Z - h(r) + \tilde{h}(\tilde{r}))$$

Polar Coordinates

$$E_{\text{out}}(\tilde{r}) = \iint \frac{1}{\lambda Q(\tilde{r}, r, \theta)} e^{2\pi i Q(\tilde{r},r,\theta)/\lambda} r d\theta dr,$$

where

$$Q(\tilde{r}, r, \theta) = \sqrt{r^2 - 2r\tilde{r}\cos\theta + \tilde{r}^2 + (h(r) - \tilde{h}(\tilde{r}))^2} + n(Z - h(r) + \tilde{h}(\tilde{r})).$$

Double integrals are hard!

Fresnel Approximation

Assuming a *large separation* between the lenses and that the lenses are themselves *thin*, we get

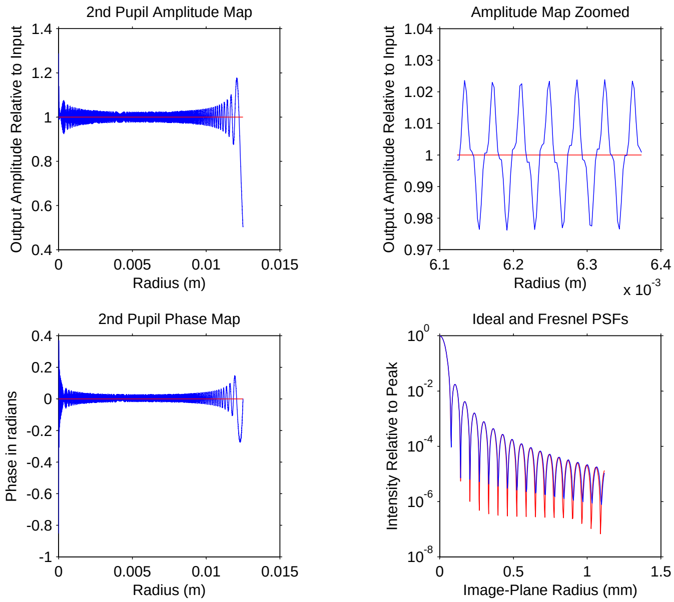
$$
\begin{aligned}
\sqrt{r^2 - 2r\tilde{r}\cos\theta + \tilde{r}^2 + (h(r) - \tilde{h}(\tilde{r}))^2} \\
\approx (h(r) - \tilde{h}(\tilde{r})) + \frac{r^2 - 2r\tilde{r}\cos\theta + \tilde{r}^2}{2z}.
\end{aligned}
$$

Fresnel uses this approximation in the exponential (and the "paraxial" approximation in the leading factor) to get the *standard Fresnel approximation*:

$$
E_{\text{out}}(\tilde{r}) = \frac{2\pi}{\lambda Z} e^{\pi i \frac{\tilde{r}^2}{z\lambda} + 2\pi i \frac{(n-1)\tilde{h}(\tilde{r})}{\lambda}} \int e^{\pi i \frac{r^2}{z\lambda} - 2\pi i \frac{(n-1)h(r)}{\lambda}} J_0(2\pi r\tilde{r}/z\lambda) r\,dr.
$$

Fresnel Examples

Flat Glass $A \equiv 1$



$n = 1.5$. $D = 25$mm. $z = 15D$. $\lambda = 632.8$nm.

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Galilean Telescope: $A \equiv a$

$$R(\tilde{r}) = a\tilde{r} \qquad \tilde{R}(r) = r/a$$

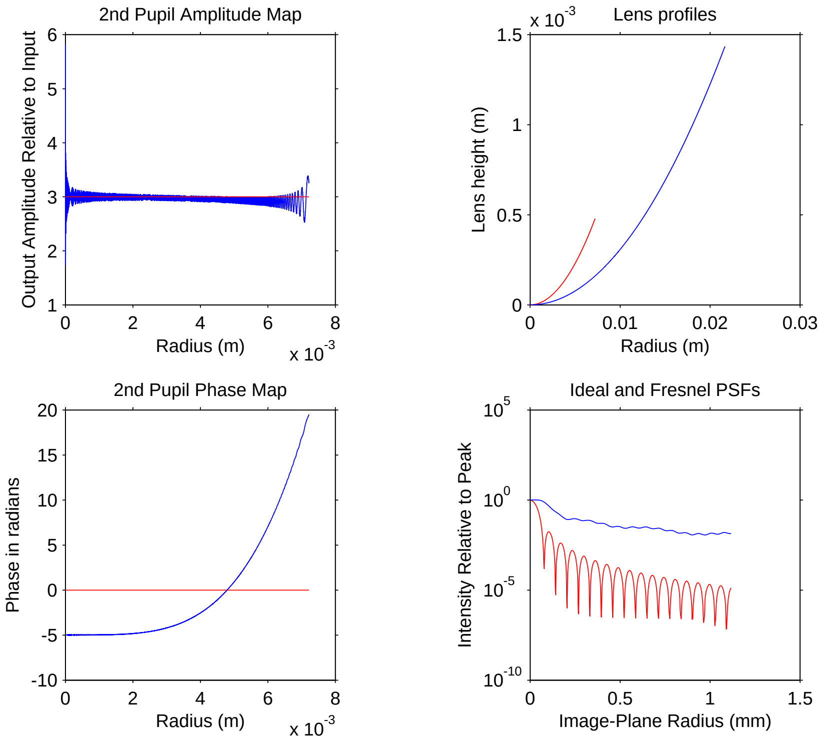
$$h(r) = z + \frac{\sqrt{Q_0^2 + (n^2-1)(1-1/a)^2 r^2} - |Q_0|}{(n^2-1)(1-1/a)}$$

$$\tilde{h}(\tilde{r}) = \frac{\sqrt{Q_0^2 + (n^2-1)(a-1)^2 \tilde{r}^2} - |Q_0|}{(n^2-1)(a-1)}$$

If $n > 1$, then both lenses are hyperbolic.

If $n < 1$, then both lenses are elliptical.

Galilean Telescope $A = 3$



Fresnel approximation is *very bad*.

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Better Than Fresnel (BTF)

We retain the paraxial approximation for the leading factor.

We can subtract an arbitrary constant from the Q in the exponent.

Let's choose to subtract $Q(\tilde{r}, R(\tilde{r}), 0)$. We compute:

$$\begin{aligned} Q(\tilde{r}, r, \theta) - Q(\tilde{r}, R(\tilde{r}), 0) &= S(\tilde{r}, r, \theta) - S(\tilde{r}, R(\tilde{r}), 0) + n(h(R(\tilde{r})) - h(r)) \\ &= \frac{S^2(\tilde{r}, r, \theta) - S^2(\tilde{r}, R(\tilde{r}), 0)}{S(\tilde{r}, r, \theta) + S(\tilde{r}, R(\tilde{r}), 0)} + n(h(R(\tilde{r})) - h(r)) \end{aligned}$$

Then, we simplify the numerator:

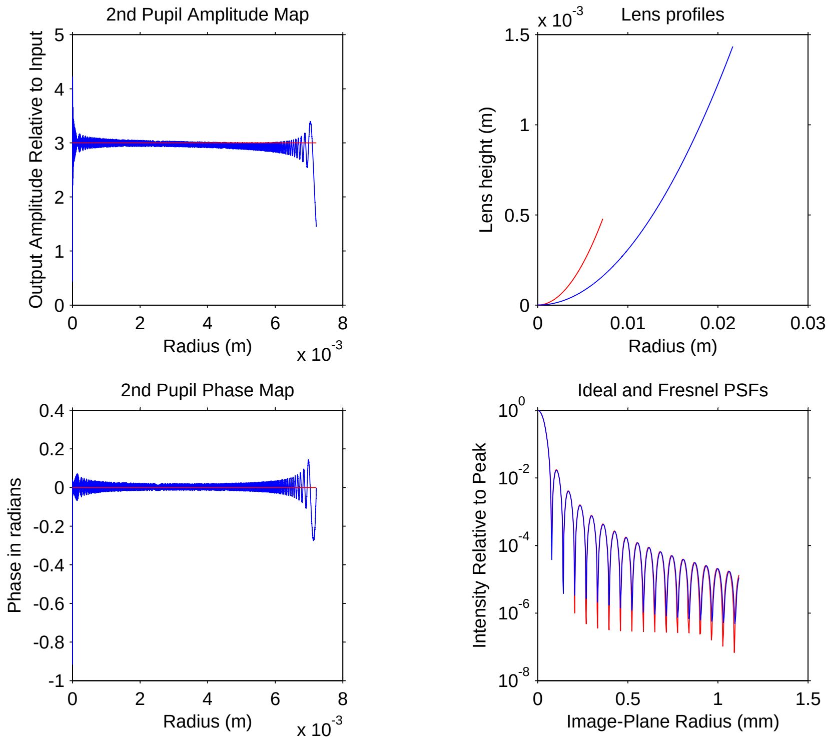
$$\begin{aligned} S^2(\tilde{r}, r, \theta) - S^2(\tilde{r}, R(\tilde{r}), 0) &= (r - R(\tilde{r}))(r + R(\tilde{r})) - 2\tilde{r}\left(r\cos\theta - R(\tilde{r})\right) \\ &\quad + \left(h(r) - h(R(\tilde{r}))\right)\left(h(r) + h(R(\tilde{r})) - 2\tilde{h}(\tilde{r})\right). \end{aligned}$$

Mo' Better

So far, everything is exact (except for the paraxial approximation). The only other approximation is to replace $S(\tilde{r}, r, \theta)$ in the denominator with $S(\tilde{r}, R(\tilde{r}), 0)$ so that the denominator becomes just $2S(\tilde{r}, R(\tilde{r}), 0)$. Replacing the integral on θ with the appropriate Bessel function, we get a new approximation:

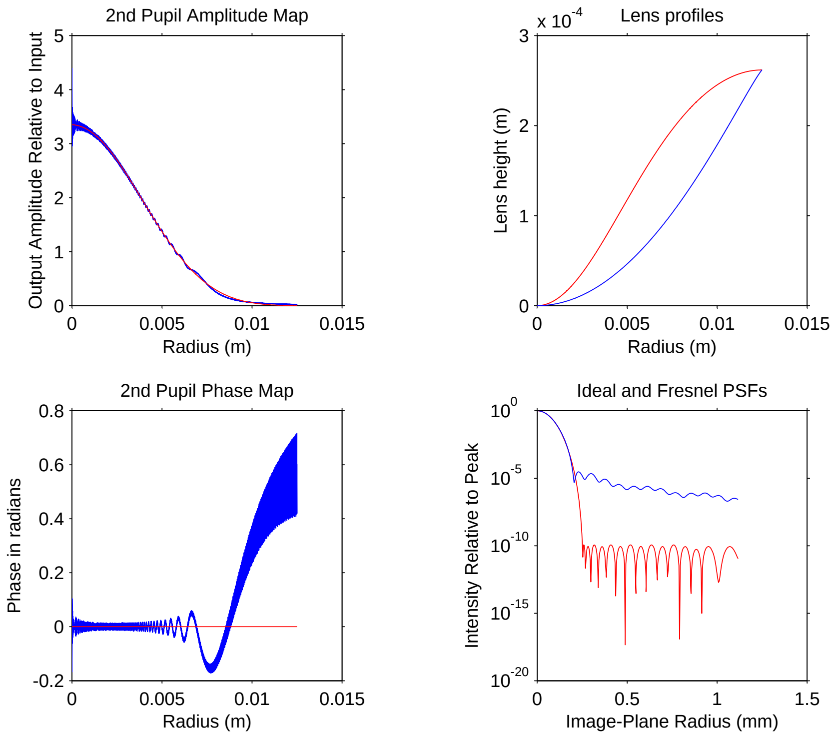
$$E_{\text{out}}(\tilde{r}) \approx \frac{2\pi}{\lambda Z} \int e^{2\pi i \left(\frac{r^2 - R(\tilde{r})^2 + 2\tilde{r}R(\tilde{r}) + \left(h(r) - h(R(\tilde{r}))\right)\left(h(r) + h(R(\tilde{r})) - 2\tilde{h}(\tilde{r})\right)}{2S(\tilde{r}, R(\tilde{r}), 0)\lambda} + \frac{n}{\lambda}\left(h(R(\tilde{r})) - h(r)\right) \right)} \cdot J_0\left(\frac{2\pi\tilde{r}r}{\lambda S(\tilde{r}, R(\tilde{r}), 0)} \right) r dr.$$

Example: $A \equiv 3$



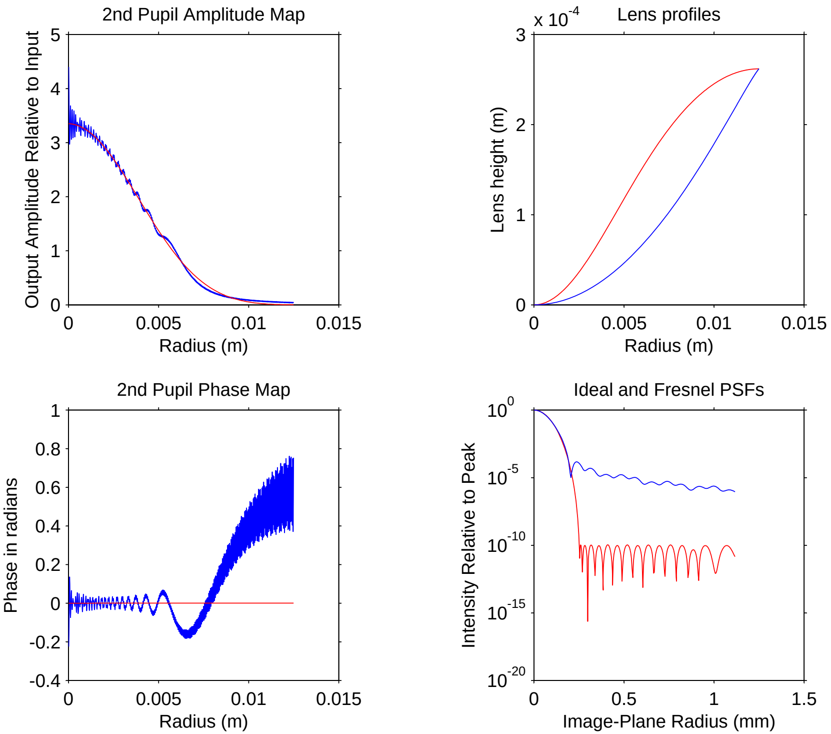
This approximation is much better than Fresnel.

Pupil Mapping for High Contrast (PIAA)



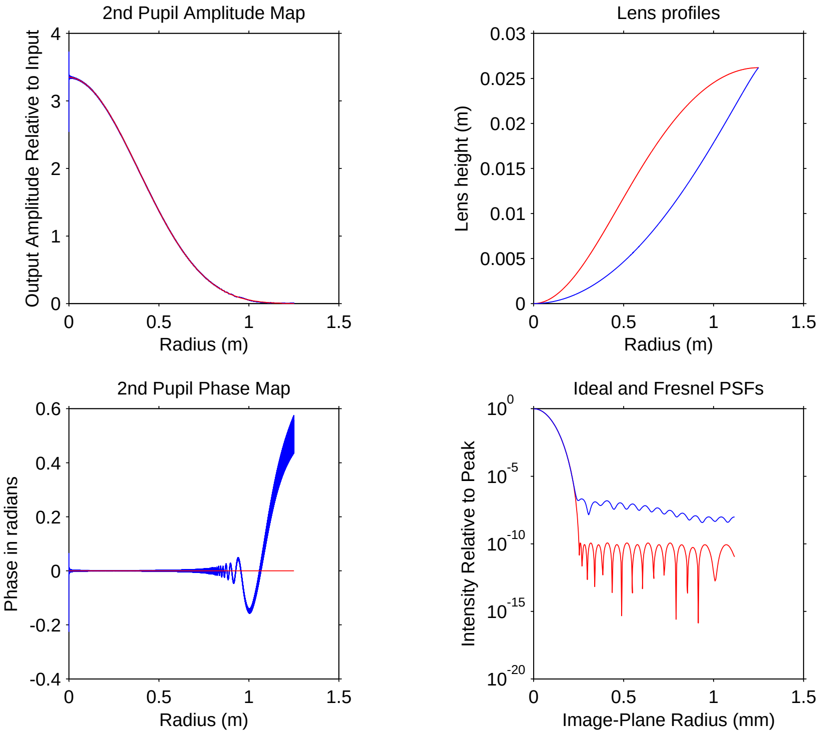
Designed for 10^{-10}. Delivers 10^{-5}.

Brute Force Huygens Wavelet Integral



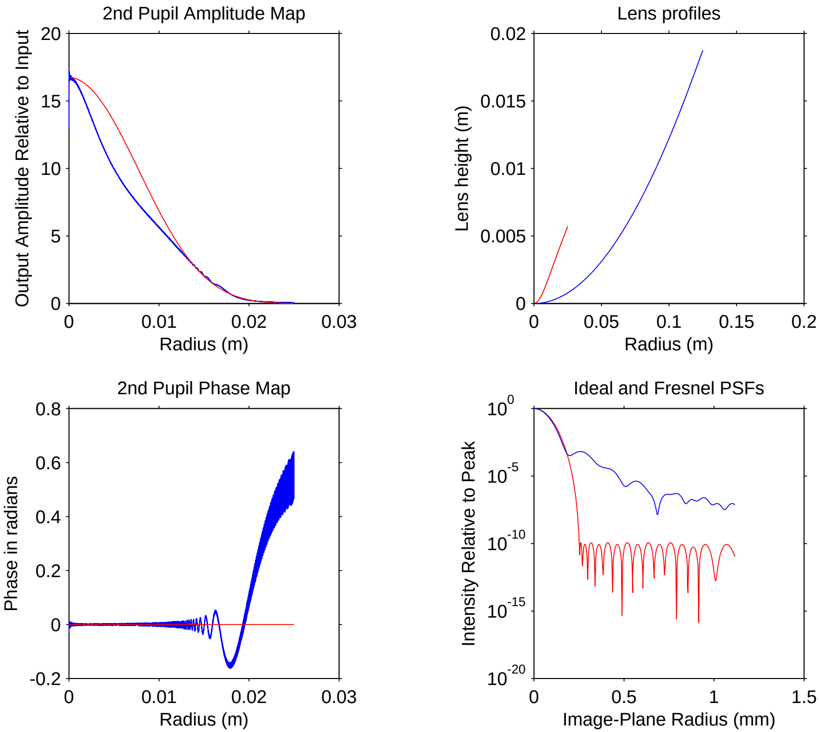
Full 2D integration: 500 r-values and 500 θ-values.
Increased the wavelength by a factor 10.
The degradation is *fundamental physics*, not numerical error.

How about larger lenses?



$D = 2.5$m (100 times bigger). Discretization: 500000 points. First sidelobe: 1.3×10^{-7}.

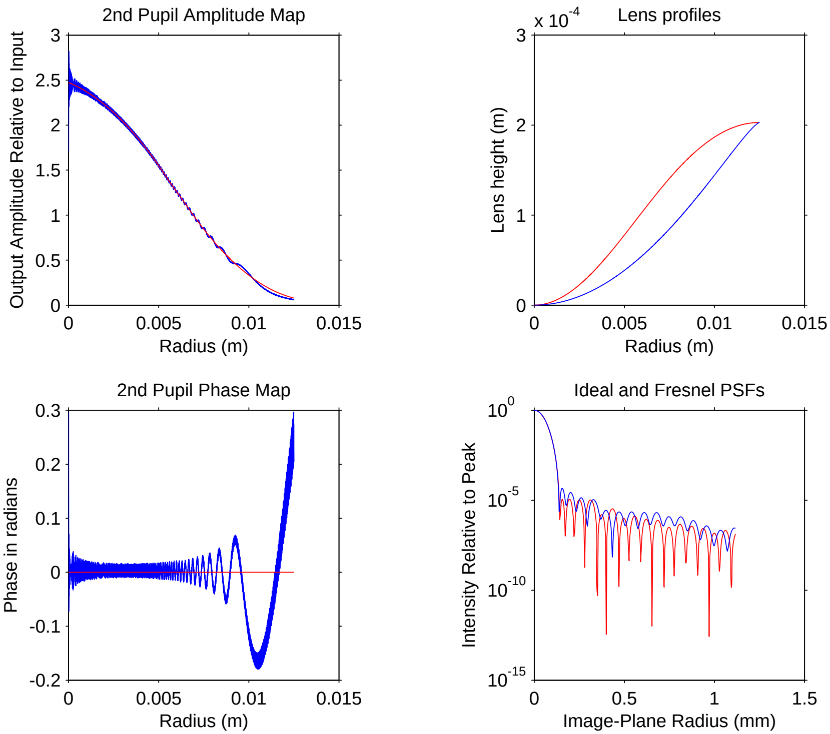
Big Primary, Small Secondary



A 10 inch primary and a 2 inch secondary.

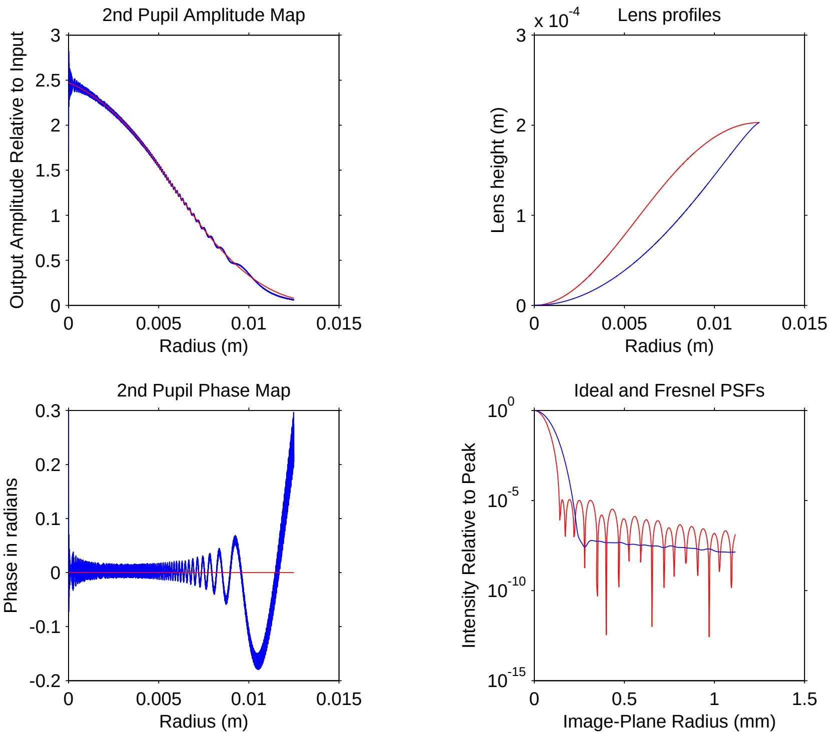
A DISASTER!

Design for Medium Contrast



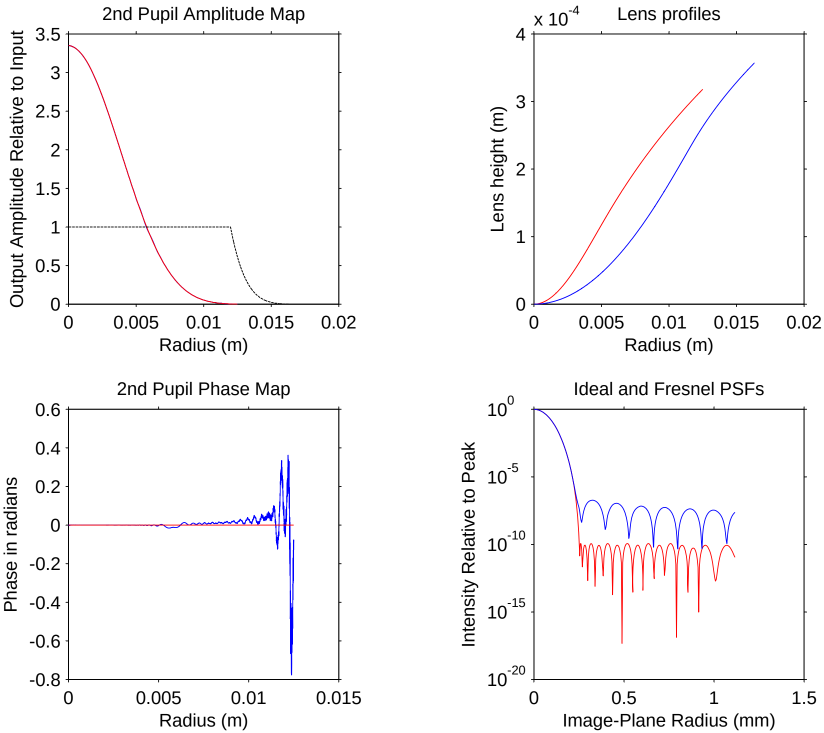
Designed for 10^{-5}. Delivers 10^{-5}.
$\max(A)$ is smaller...less off-axis distortion to correct.

Medium Contrast Pupil Mapping with Back-End Apodizer



Hybrid system designed for 10^{-10}. Delivers $10^{-7.5}$.

Front-End Apodizer



This hybrid system is about as good as the back-end apodizer.
Easier to manufacture.