Shape Optimization in Designing the "Terrestrial Planet Finder" Telescope

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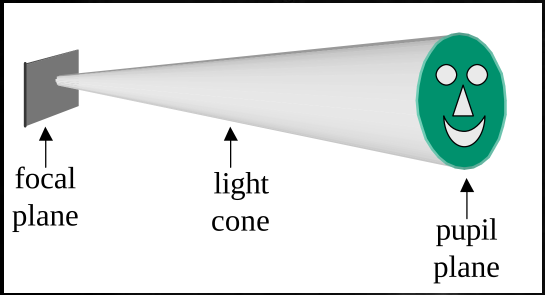
A Terrestrial Planet Finder Telescope

The problem is to design and build a space telescope that will be able to "see" planets around nearby stars (other than the Sun).

Consider a telescope. Light enters the front of the telescope. This is called the *pupil plane*.

The telescope focuses all the light passing through the pupil plane from a given direction at a certain point on the *focal plane*, say $(0, 0)$.



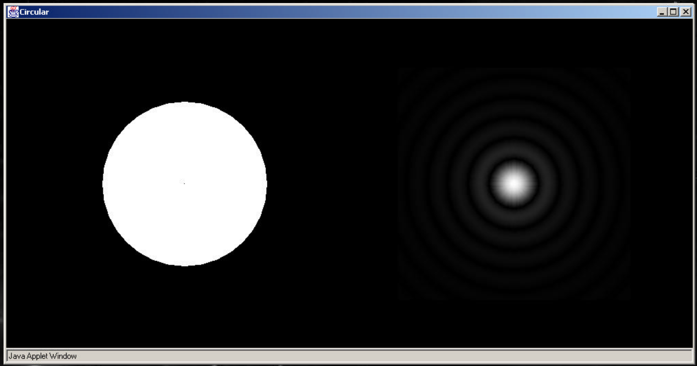
However, the wave nature of light makes it impossible to concentrate all of the light at a point. Instead, a small disk, called the *Airy disk*, with diffraction rings around it appears.

These diffraction rings are bright relative to any planet that might be orbiting a nearby star and so would completely hide the planet. The Sun, for example, would appear 10^{10} times brighter than the Earth to a distant observer.

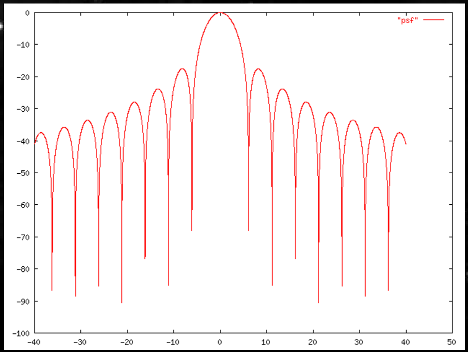
By placing a mask over the pupil, one can design the shape and strength of the diffraction rings. The problem is to find an optimal shape so as to put a very deep *null* very close to the Airy disk.

Airy Disk and Diffraction Rings

A conventional telescope has a circular openning as depicted by the left side of the figure. Visually, a star then looks like a small disk with rings around it, as depicted on the right.

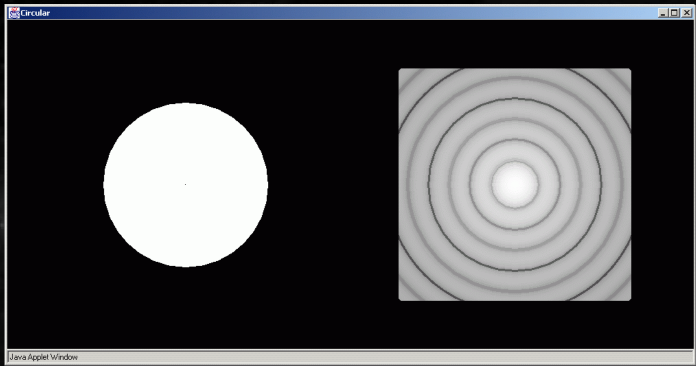


The rings grow progressively dimmer as this log-plot shows:



Airy Disk and Diffraction Rings—Log Scaling

Here's the same Airy disk from the previous slide plotted using a logarithmic brightness scale with $10^{-11} = -110$dB set to black:



The problem is to find an aperture mask, i.e. a pupil plane mask, that yields a -100 dB *null* somewhere near the first diffraction ring. *A hard problem!* Such a null would appear almost black in this log-scaled image.

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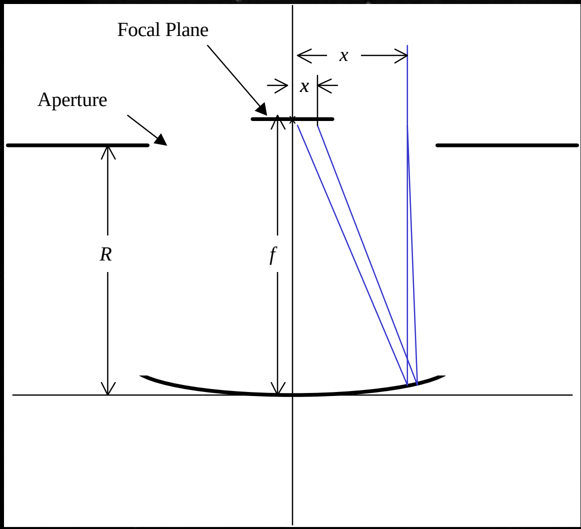
Diffraction Limited Optics

A coherent plane wave passing the pupil plane produces an interference pattern at the focal plane given by

$$E(\xi, \zeta) = \iint e^{-ik\Delta_{x,y}(\xi,\zeta)} dy dx$$



where the double integral extends over the open portion of the mask, $k = (2\pi\lambda)^{-1}$ is the wave number, and $\Delta_{x,y}(\xi, \zeta)$ denotes the difference in the distance from the point (x, y) on the pupil plane to $(0, 0)$ and from (x, y) to (ξ, ζ). The formula for $\Delta_{x,y}(\xi, \zeta)$ is complicated but is well approximated by its linear part:

$$\Delta_{x,y}(\xi, \zeta) \approx (x\xi + y\zeta)/f$$

Multiple Pupils: Electric Field

Consider an aperture mask consisting of m pupils (opennings) where the j-th pupil is given by

$$\{(x,y) : -\frac{1}{2} \le x \le \frac{1}{2}, A_j(x) \le y \le B_j(x)\}.$$

We only consider masks that are symmetric with respect to both the x and y axes. Hence, the functions $A_j()$ and $B_j()$ are even functions which we define only for the upper halfplane and explicitly negate for the lower half plane.

In such a situation, the electric field $E(\xi,\zeta)$ is real and also symmetric about both the x and y axes. It is given by

$$\begin{aligned} E(\xi,\zeta) &= \sum_j \int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\int_{A_j(x)}^{B_j(x)} e^{i(x\xi+y\zeta)} dy + \int_{-B_j(x)}^{-A_j(x)} e^{i(x\xi+y\zeta)} dy \right) dx \\ &= 4 \sum_j \int_0^{\frac{1}{2}} \cos(x\xi) \frac{\sin(B_j(x)\zeta) - \sin(A_j(x)\zeta)}{\zeta} dx \end{aligned}$$

The intensity of the light at (ξ,ζ) is given by the square of the electric field.

Optimization Problem

Because of the symmetry, we only need to optimize in the first quadrant:

$$\text{maximize } 4\sum_j \int_0^{\frac{1}{2}} (B_j(x) - A_j(x))dx$$

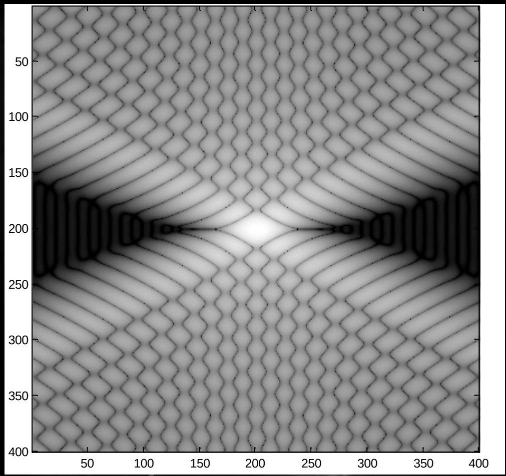
$$\begin{aligned} \text{subject to } & -10^{-5}E(0,0) \le E(\xi,\zeta) \le 10^{-5}E(0,0) && (\xi,\zeta) \in \mathcal{O} \\ & 0 \le A_j(x) \le B_j(x) \le 1/2 && j = 1,\ldots,m,\ \forall x \\ & B_j(x) \le A_{j+1}(x) && j = 1,\ldots,m,\ \forall x \end{aligned}$$

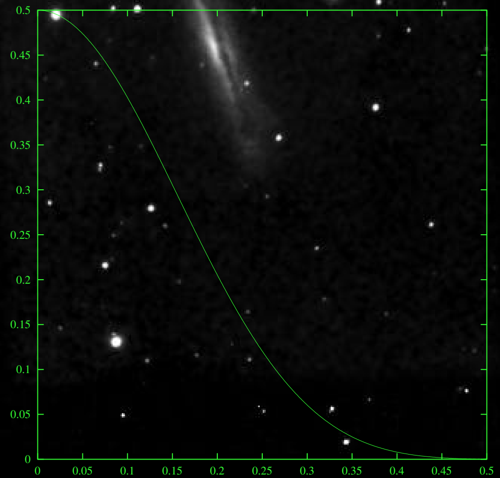
The objective function is the total open area of the mask. The first constraint guarantees 10^{-10} light intensity throughout a desired region of the focal plane, and the remaining constraints ensure that the mask is really a mask.

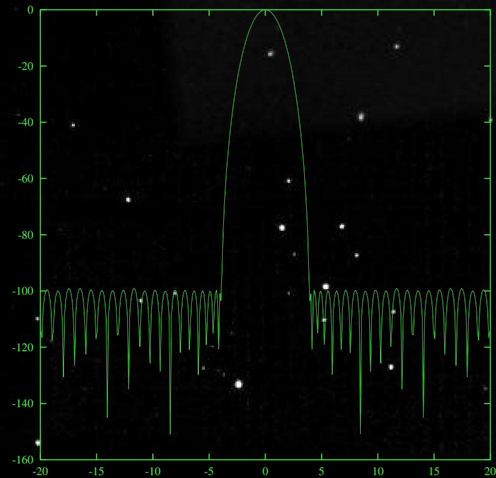
If the set $\mathcal{O}$ is a subset of the x-axis, then the problem is entirely linear (a linear programming problem).

One Pupil w/ On-Axis Constraints

$\mathcal{O} = \{(\xi, 0) : \xi_0 \leq \xi \leq \xi_1\}$
$\xi_0 = 4$
$\xi_1 = 40$
Log scale:
white $=$ 0dB,
black $= -110$dB
Thruput $= 37\%$







One Pupil w/ Triangle Constraints

$\mathcal{O}$ = triangle w/ vertices: $(\xi_0, 0)$, $(\xi_0 + 17, 0)$, $(\xi_0 + 17, 17)$

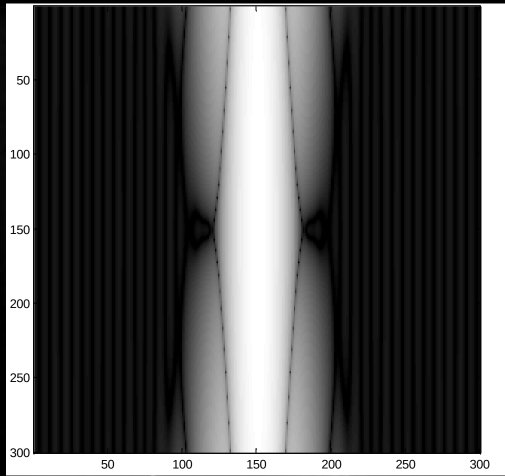
$\xi_0 = 4$

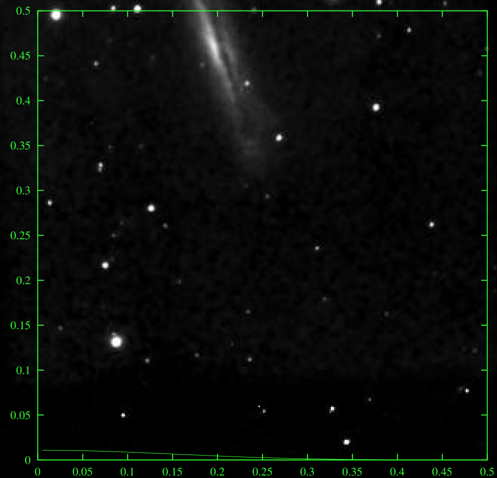
Log scale:

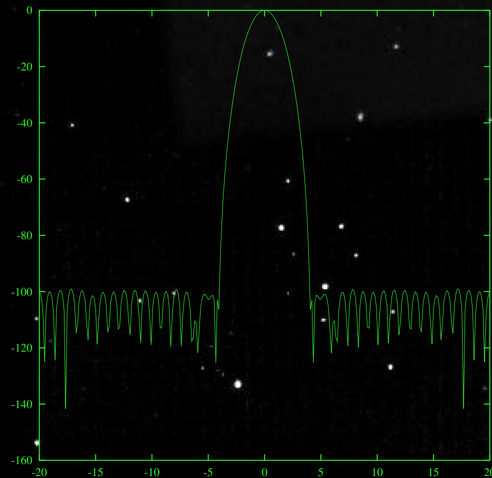
white = 0dB,

black = -110dB

Thruput = 0.8%







Two Pupils w/ Triangle Constraints

$\mathcal{O}$ = triangle w/ vertices: $(\xi_0, 0)$, $(\xi_0 + 17, 0)$, $(\xi_0 + 17, 17)$

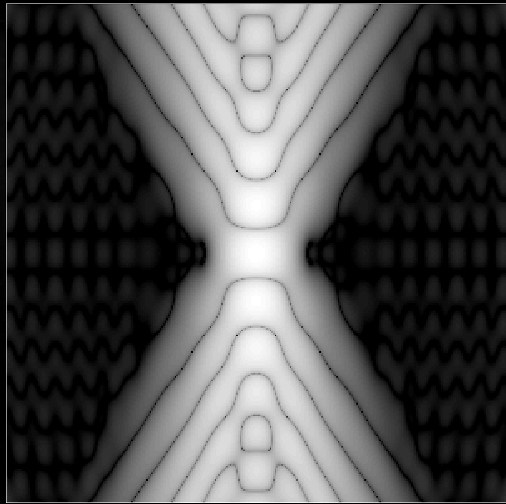
$\xi_0 = 4$

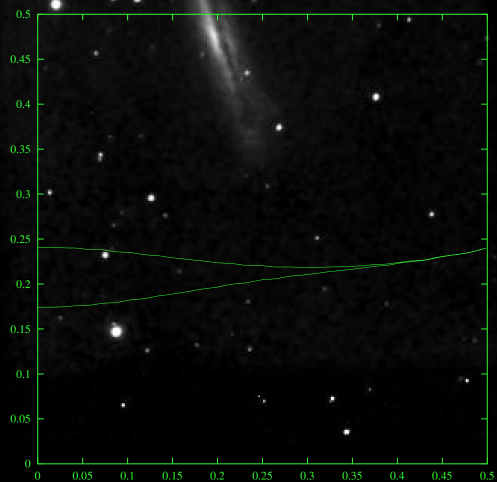
Log scale:

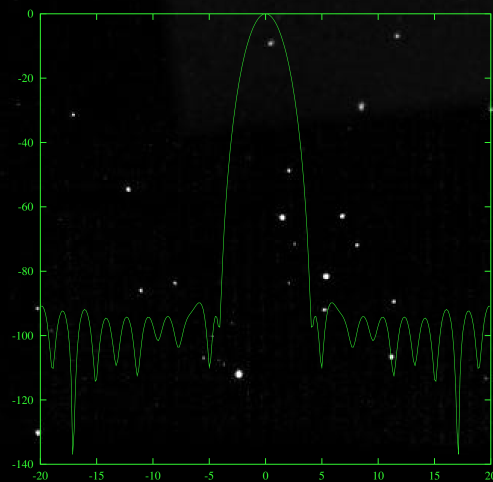
white = 0dB,

black = -110dB

Thruput = 5.9%







Four Pupils w/ Triangle Constraints

$\mathcal{O}$ = triangle w/ vertices: $(\xi_0, 0)$, $(\xi_0 + 17, 0)$, $(\xi_0 + 17, 17)$

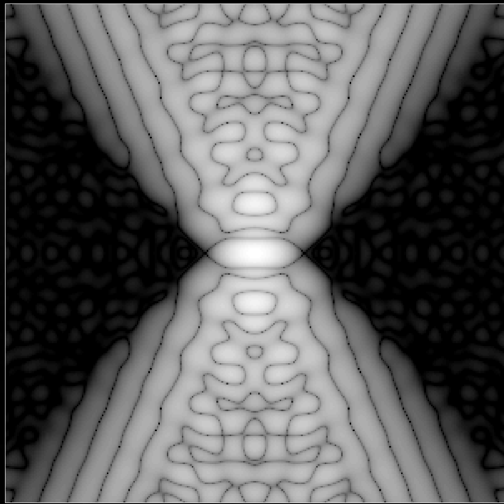
$\xi_0 = 4$

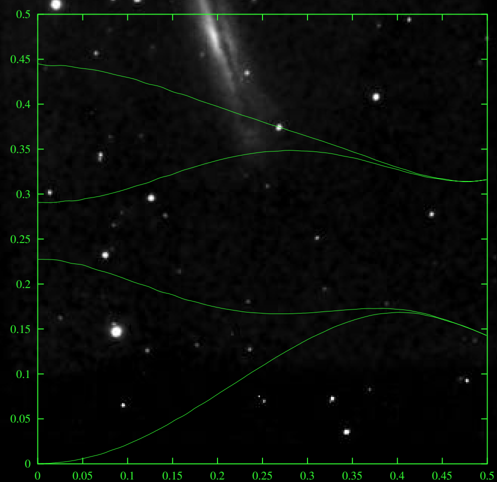
Log scale:

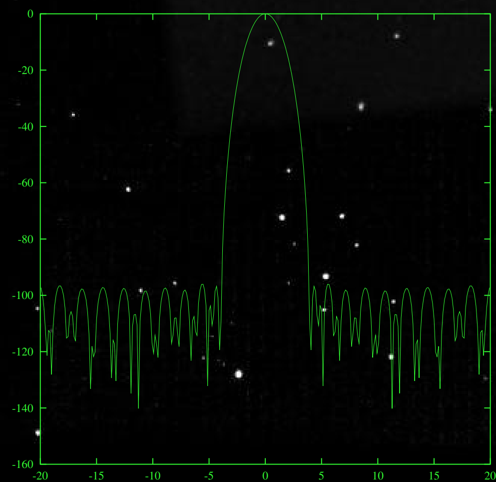
white = 0dB,

black = −110dB

Thruput = $\mathbf{28.3\%}$







Six Pupils w/ Triangle Constraints

$\mathcal{O}$ = triangle w/ vertices: $(\xi_0, 0)$, $(\xi_0 + 17, 0)$, $(\xi_0 + 17, 17)$

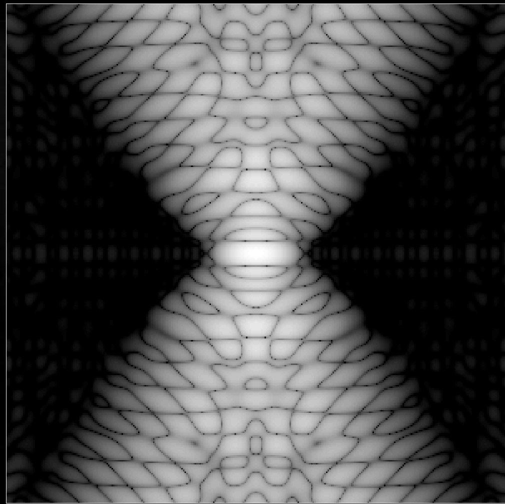
$\xi_0 = 4$

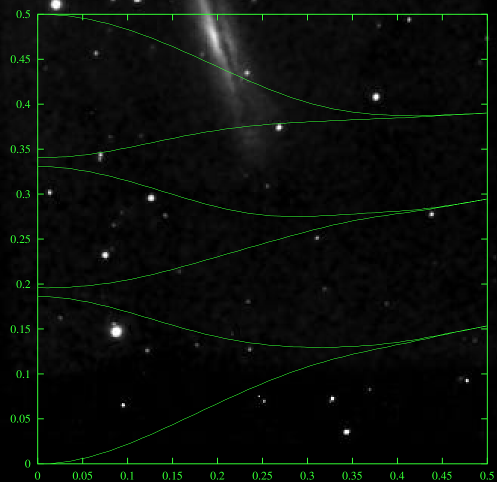
Log scale:

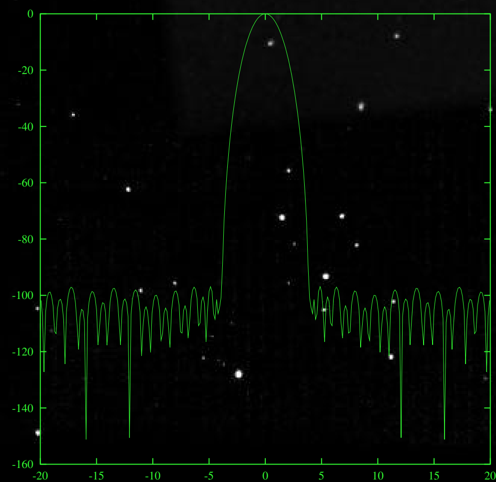
white = 0dB,

black = -110dB

Thruput = 36.0%







Six Pupils, Eclipse

$\mathcal{O}$ = triangle w/ vertices: $(\xi_0, 0)$, $(\xi_0 + 17, 0)$, $(\xi_0 + 17, 17)$

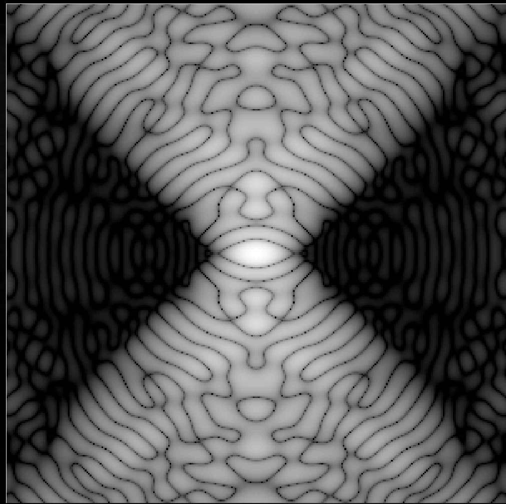
$\xi_0 = 3.5$

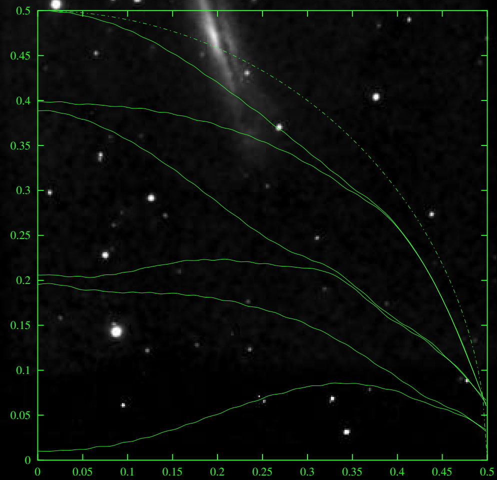
Log scale:

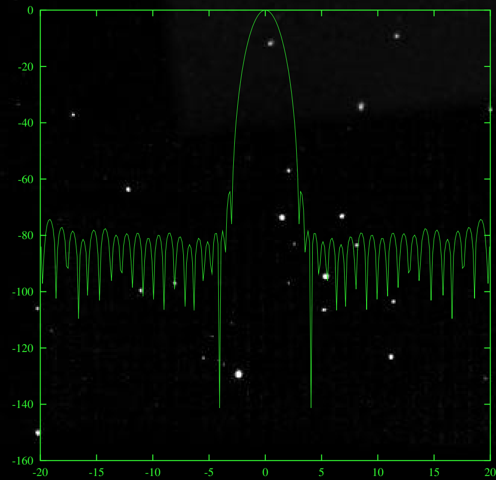
white = 0dB,

black = −90dB

Thruput = 39.6%







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Four Pupils, Ellipse

$\mathcal{O}$ = triangle w/ vertices: $(\xi_0, 0)$, $(\xi_0 + 17, 0)$, $(\xi_0 + 17, 17)$

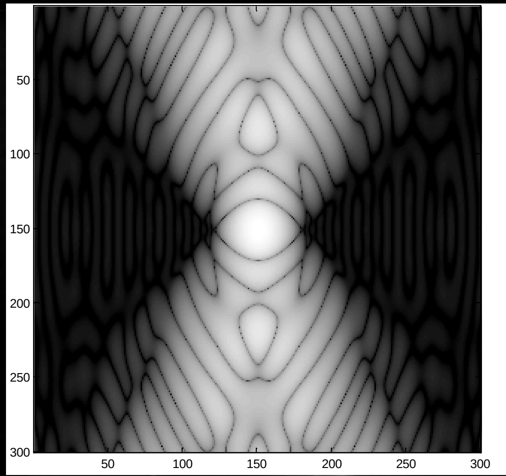
$\xi_0 = 4$

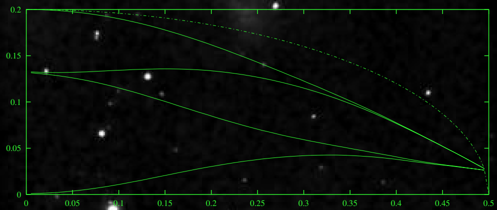
Log scale:

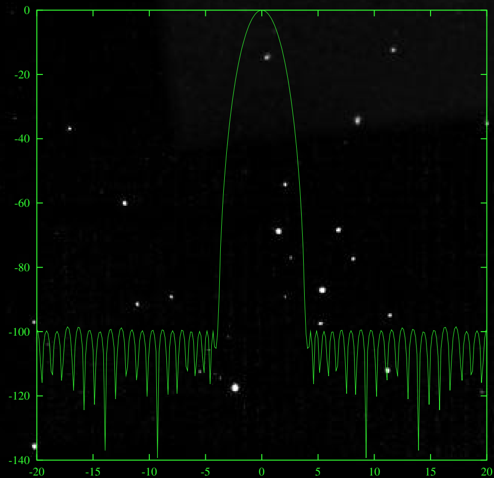
white = 0dB,

black = -110dB

Thruput = 37.0%



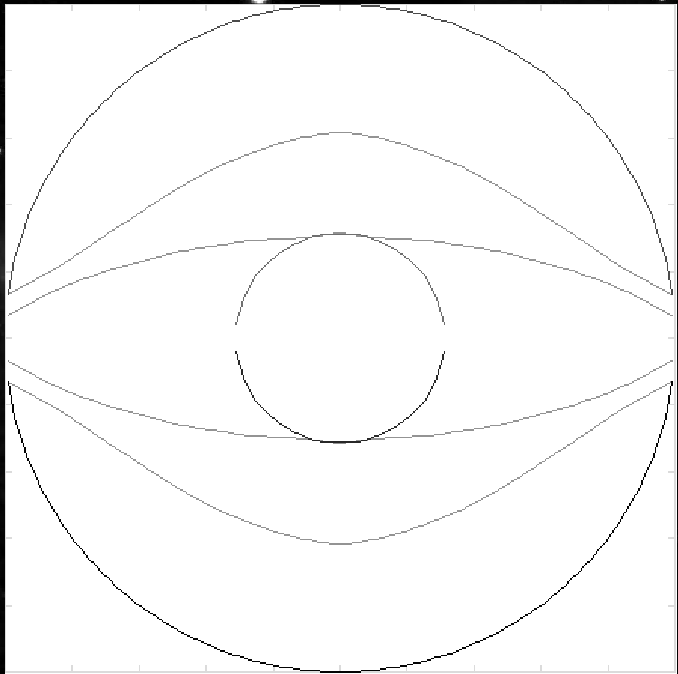




Field Test

Dim Double Splitter Mask

A mask was made for a 3.5" Questar. The mask was cut from paper with scissors (a crude tool at best) according to the template shown, backed with cardboard, and framed with 4" PVC endcap.



The outer circle represents the full aperture, the inner circle the central obstruction, and the remaining arcs the mask openning.

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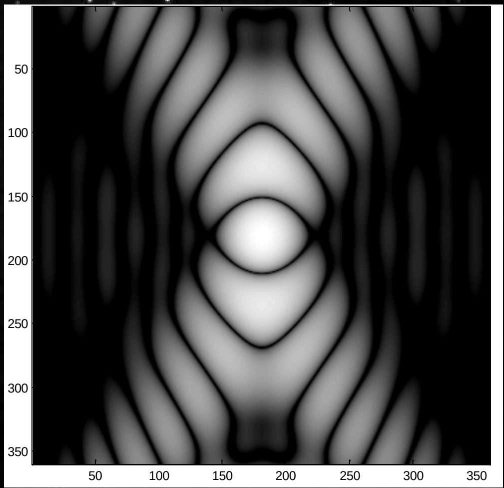
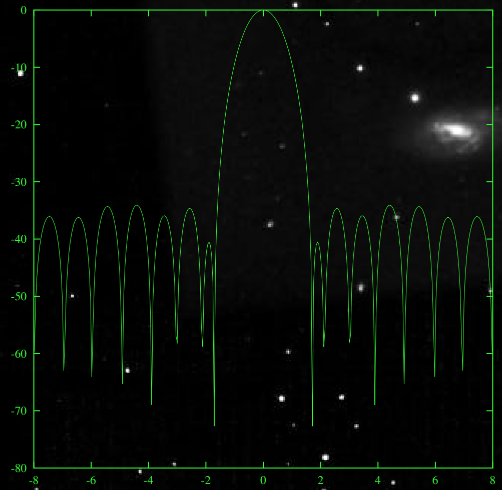
Computed PSF

Logarithmically scaled plot of the 2-D point spread function and a graph of its x-axis slice. White is 0 dB and black is -40 dB. Throughput is 18.2%.



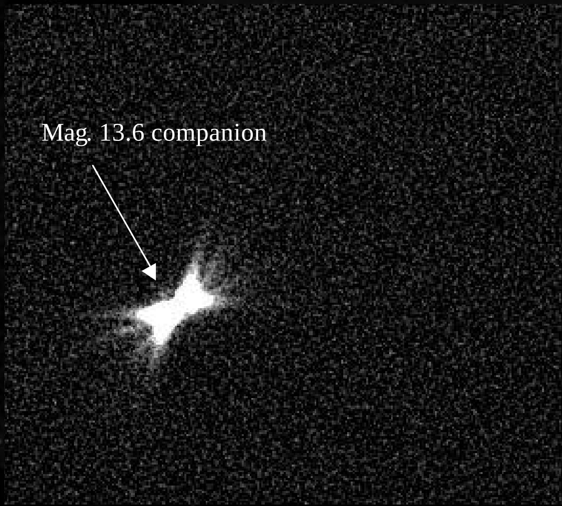


31 Leonis

31 Leonis is a dim double.
Primary/secondary visual magnitude: $\mathbf{4.37/13.6}$
Luminance difference $= \mathbf{9.2} = \mathbf{-36.8}$ dB
Separation: $\mathbf{7.9''} = \mathbf{6.9\lambda/D}$ (at $\mathbf{500}$nm). Position Angle: $\mathbf{44^\circ}$

Without mask:

With mask:



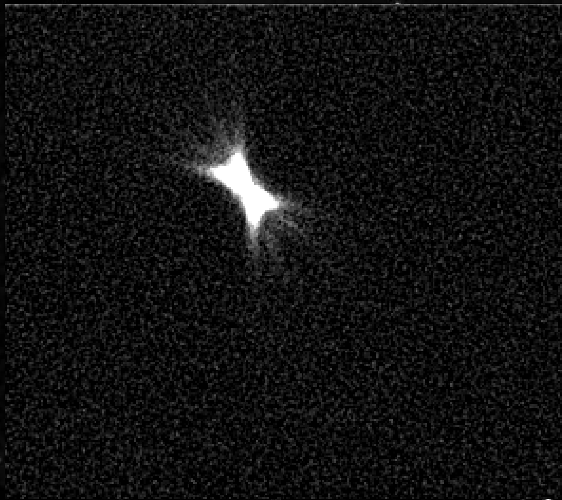
The secondary is to the upper left of the primary in the mask image.

Is it real?

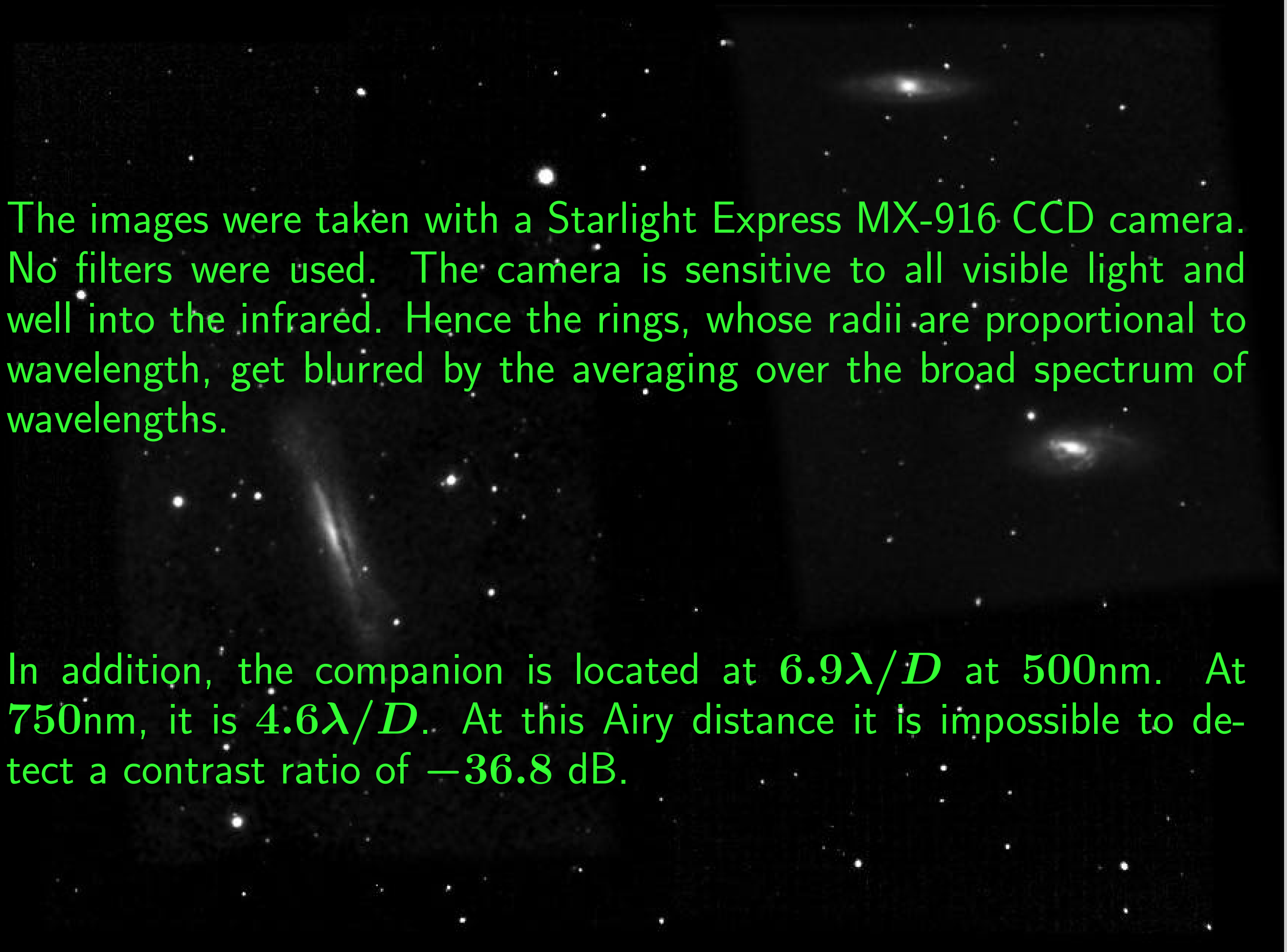
We took another image with the mask rotated about 90°. The rotated mask shows no hint of a secondary:

Original orientation:

Rotated:



Where are the diffraction rings?



The images were taken with a Starlight Express MX-916 CCD camera. No filters were used. The camera is sensitive to all visible light and well into the infrared. Hence the rings, whose radii are proportional to wavelength, get blurred by the averaging over the broad spectrum of wavelengths.

In addition, the companion is located at $6.9\lambda/D$ at 500nm. At 750nm, it is $4.6\lambda/D$. At this Airy distance it is impossible to detect a contrast ratio of -36.8 dB.

Circularly Symmetric Apodization

Instead of a mask, we consider now a circularly symmetric apodized aperture:

$$E(\xi,\zeta) = \int_0^{a/2}\int_{-\pi}^{\pi} A(r) e^{-ik(x\xi+y\zeta)/f} r\,d\theta\,dr$$

where, of course, $x = r\cos\theta$ and $y = r\sin\theta$.

Without loss of generality, we may assume that $\zeta = 0$. Hence, we look at

$$\begin{aligned} E(\xi,0) &= \int_0^{a/2} rA(r)\left(\int_{-\pi}^{\pi} e^{-i\frac{k\xi}{f}r\cos\theta}\right) d\theta\,dr \\ &= \int_0^{a/2} 2\pi r A(r) J_0\left(kr\xi/f\right) dr \end{aligned}$$

Apodization Optimization Problem

The apodization function is found by solving the following optimization problem:

$$\text{minimize} \quad \int_{\xi_0}^{\xi_1} |E(\xi,0)|^2 d\xi$$

$$\text{subject to:} \quad 2\pi \int_0^{a/2} A(r) r dr \geq \alpha \pi a^2/4$$

$$0 \leq A(r) \leq 1$$

where

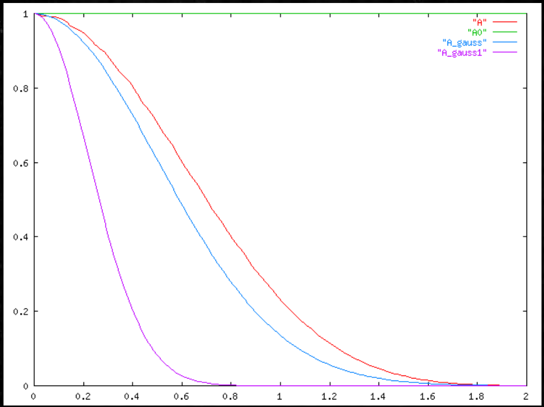
$$E(\xi,0) = \int_0^{a/2} 2\pi r A(r) J_0\left(kr\xi/f\right) dr$$

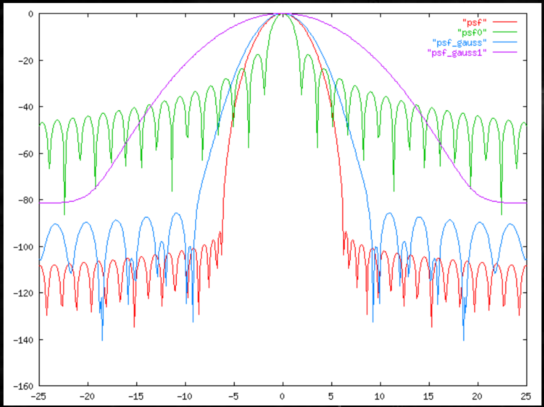
and α denotes a lower bound on the open area of the mask.

Note: we are now using a throughput (i.e., area) constraint.

Optimal vs. Gaussian Apodization

We compared with a full clear aperture (i.e., a circular-shaped aperture) and two Gaussian masks:





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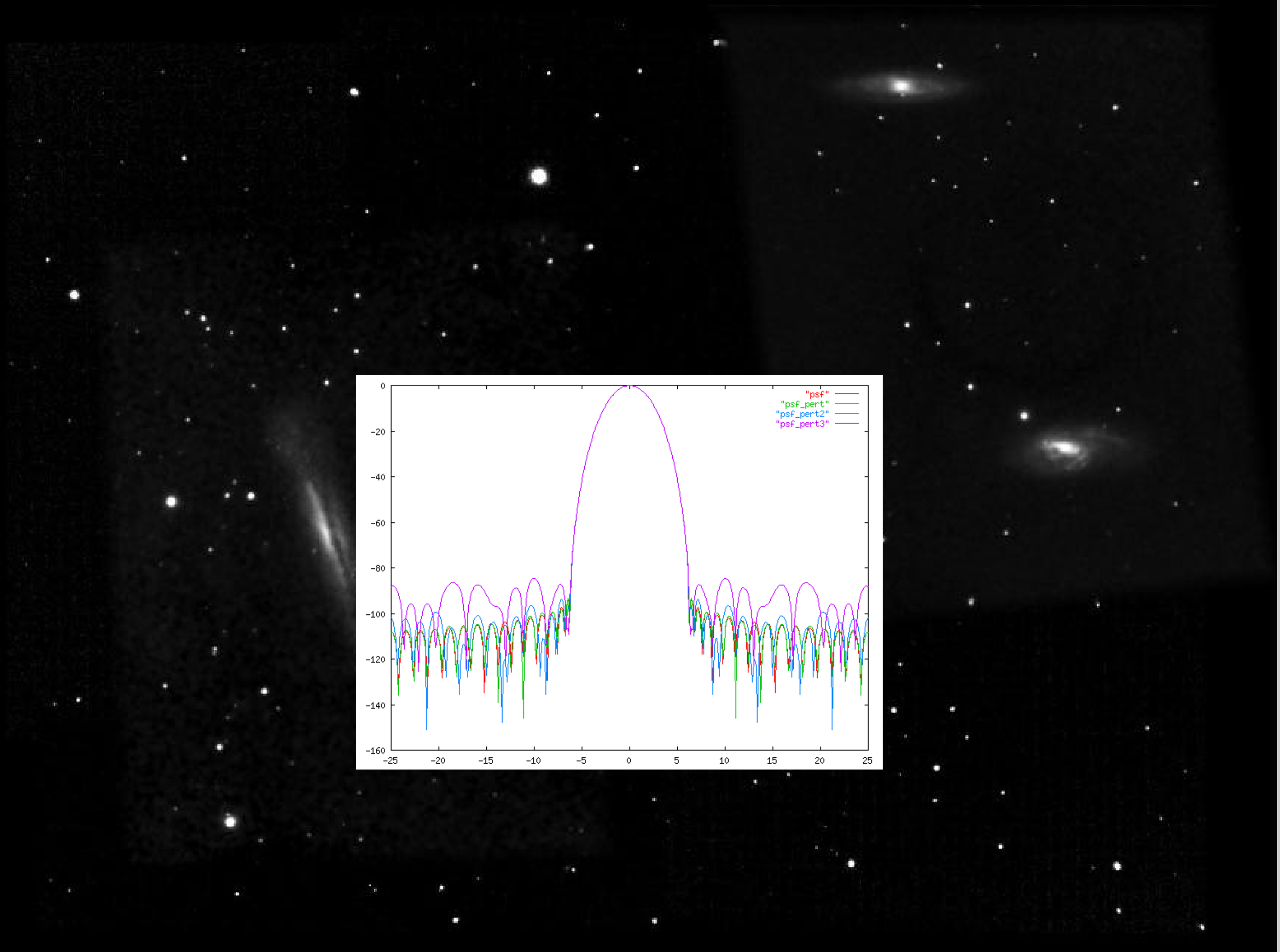
Optimal Solution vs. Perturbations



Circularly Symmetric Masks

Circularly symmetric masks can be thought of as apodizations in which the apodizing function $A(r)$ is zero-one valued.

Let

$$A(r) = \begin{cases} 1 & r_{2j} \le r \le r_{2j+1}, \qquad j = 0, 1, \ldots, m-1 \\ 0 & \text{otherwise,} \end{cases}$$

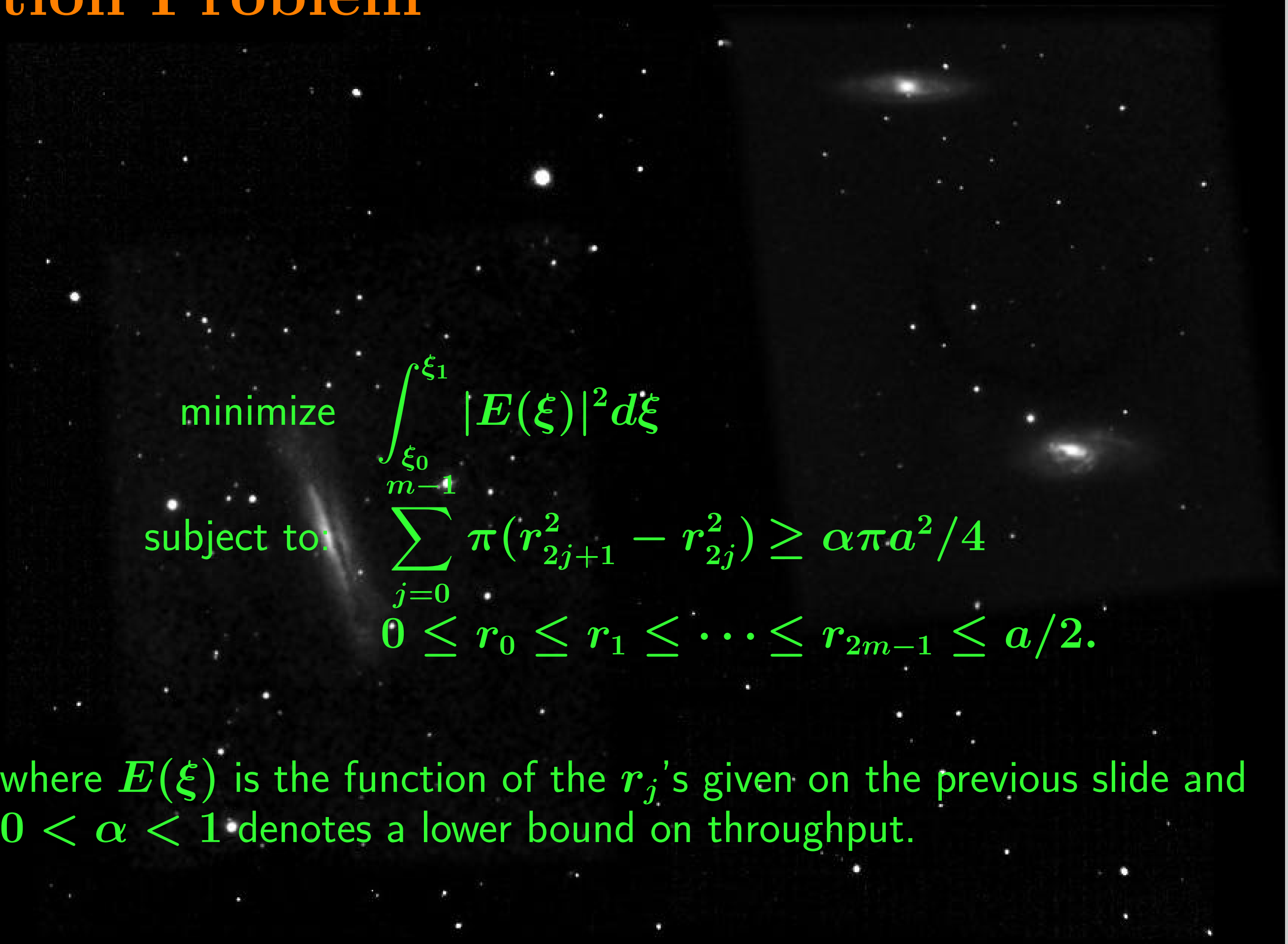
where

$$0 \le r_0 \le r_1 \le \cdots \le r_{2m-1} \le a/2.$$

We then get

$$\begin{aligned} E(\xi) &= \int_0^{a/2} 2\pi r A(r) J_0\left(2\pi k \xi r / f\right) dr \\ &= \sum_{j=0}^{m-1} \int_{r_{2j}}^{r_{2j+1}} 2\pi r J_0\left(2\pi k \xi r / f\right) dr \\ &= \sum_{j=0}^{m-1} \frac{f}{k\xi} \left(r_{2j+1} J_1\left(2\pi k \xi r_{2j+1} / f\right) - r_{2j} J_1\left(2\pi k \xi r_{2j} / f\right) \right). \end{aligned}$$

Circularly Symmetric Masks Optimization Problem



$$\text{minimize} \quad \int_{\xi_0}^{\xi_1} |E(\xi)|^2 d\xi$$

$$\text{subject to:} \quad \sum_{j=0}^{m-1} \pi (r_{2j+1}^2 - r_{2j}^2) \geq \alpha \pi a^2/4$$

$$0 \leq r_0 \leq r_1 \leq \cdots \leq r_{2m-1} \leq a/2.$$

where $E(\xi)$ is the function of the r_j's given on the previous slide and $0 < \alpha < 1$ denotes a lower bound on throughput.

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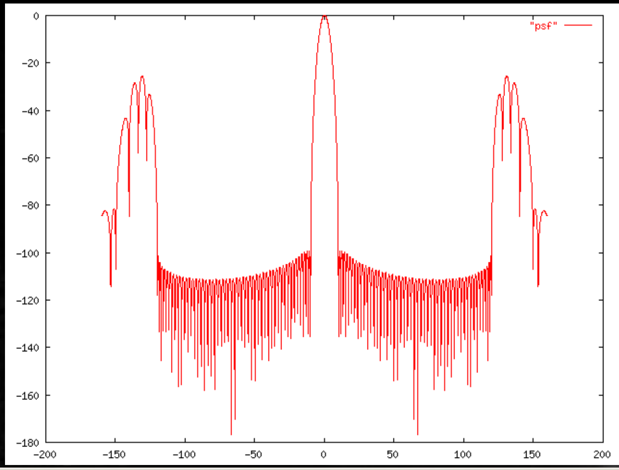
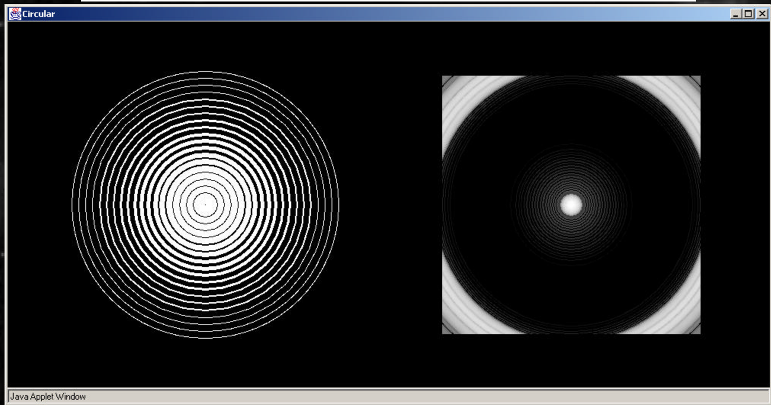
$\xi_0 = 4\lambda f/a = 10\mu$
$\xi_1 = 48\lambda f/a = 120\mu$
26 rings
x-axis scale is in microns
Airy disk and diffraction rings are on a log scale:
white $= 0$dB,
black $= -110$dB.
thruput $= 17\%$





Note: This is not a Fresnel zonal plate. We still have a mirror to focus the light. This mask is just for controlling the diffraction rings.

$\xi_0 = 4\lambda f/a = 10\mu$

$\xi_1 = 32\lambda f/a = 60\mu$

18 rings

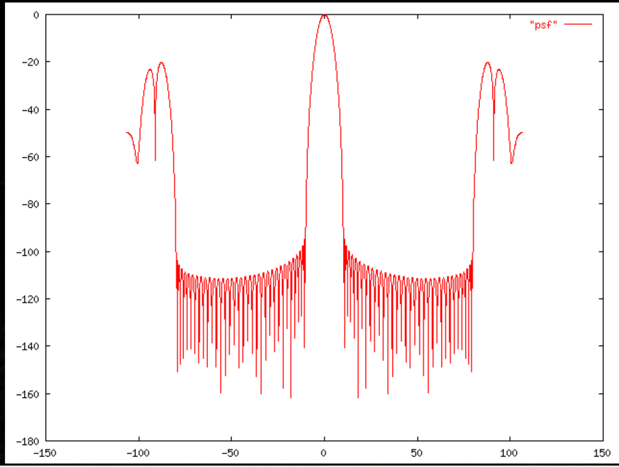
x-axis scale is in microns

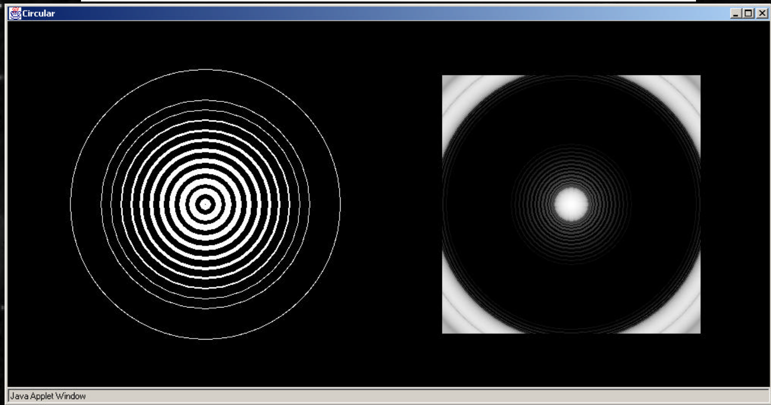
Airy disk and diffraction rings are on a log scale:

white $= 0$dB,

black $= -110$dB.

thruput $= 10\%$





Note: Fraction of the star light beyond the inner working distance: 34%.

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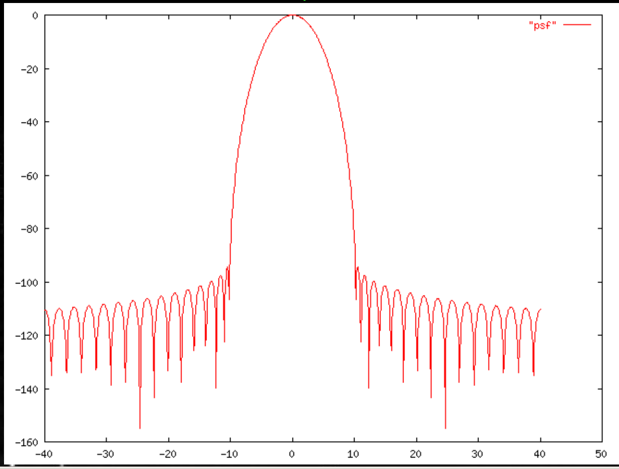
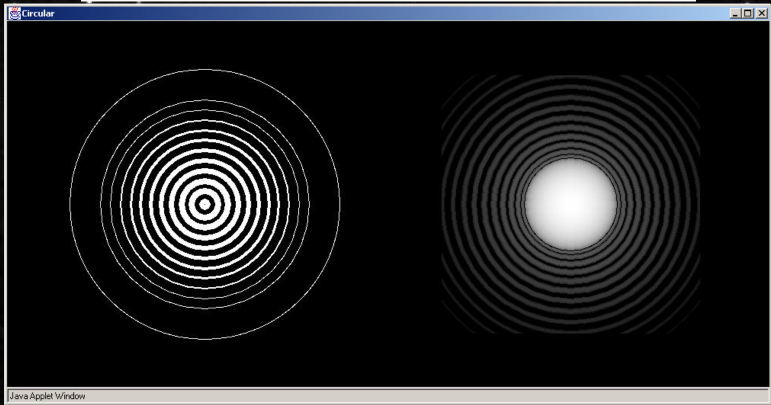
Same scenario, closer in view:





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Adding a Central Obstruction

To add a central obstruction, just give a lower bound on r_0.

Central obstruction: 18%

$\xi_0 = 7\lambda f/a = 17.5\mu$

$\xi_1 = 84\lambda f/a = 210\mu$

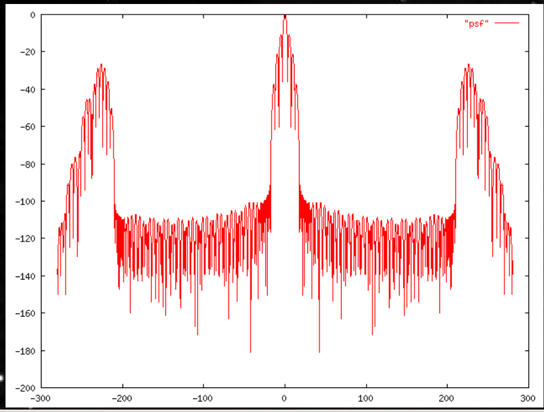
x-axis scale is in microns

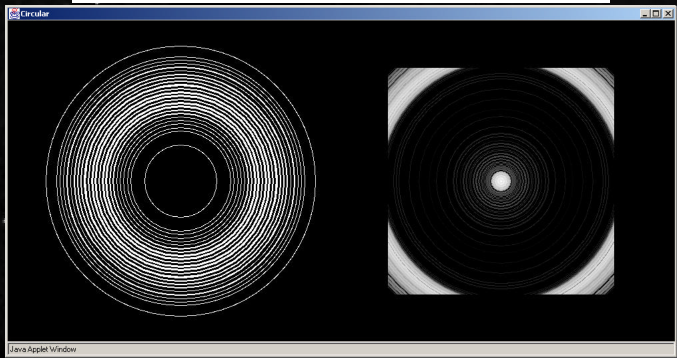
Airy disk and diffraction rings are on a log scale:

white $= 0$dB,

black $= -110$dB.

thruput $= 19\%$

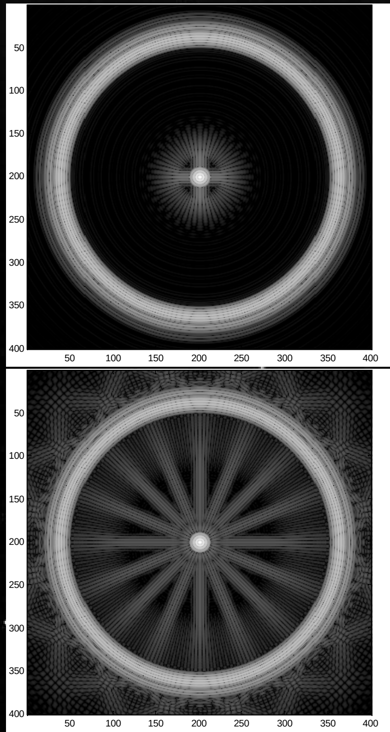




Using Spider Vanes to Support the Rings

4 Vanes w/ $h = 0.004$ vs. 16 Vanes w/ $h = 0.001$

Terms in infinite Jacobi-Anger expansion: 7



4 Vanes w/ $h = 0.001$ vs. 16 Vanes w/ $h = 0.00025$

Central obstruction: 18%
x-axis scale is in microns
$\xi_0 = 7\lambda f/a = 17.5\mu$
$\xi_1 = 84\lambda f/a = 210\mu$
Airy disk and diffraction rings are on a log scale:
white = 0dB,
black = -110dB.
thruput = 19% Terms in infinite expansion: 7
Airy disk and diffraction rings are on a log scale:
white = 0dB,
black = -110dB.