Chapter 1

Pupil Plane Amplitude Coronagraphy

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Unlike in a classical Lyot coronagraph, where the starlight is blocked at the image plane to remove it entirely from the optical system, the principle behind pupil plane coronagraphy is to reshape the PSF so that it contains regions of high-contrast. In amplitude coronagraphs this is done by changing the magnitude of the electric field as it passes through the entrance pupil of the system. This can be done through apodization, where the transmission is a smooth function of position in the pupil, or through an actual change in shape of the aperture, what is called a binary transmission function, so that the resulting point spread function is no longer the Airy function associated with an open circular aperture but rather reflects the more intricate open areas of the pupil. In this chapter, we describe the mathematics behind pupil plane amplitude coronagraphy and describe various optimal approaches to designing coronagraphs.

1. Statement of the Problem

We consider the simplest optical system consisting of an entrance pupil described by a set S with an apodization function $A(x, y)$, where (x, y) are coordinates across the pupil and $A(x, y)$ takes on values between 0 and 1 and represents the transmission of the pupil. Thus, for a square aperture for example, S would be given by:

$$S = \{(x, y) : -D/2 \leq x \leq D/2, -D/2 \leq y \leq D/2\}.$$

For a circular aperture S would be given by

$$S = \left\{(x, y) : \sqrt{x^2 + y^2} \leq D/2\right\}$$

where D is the width/diameter of the aperture.

Assuming a uniform electric field of wavelength λ and amplitude E_0 at the pupil followed by a lens of focal length f, and using only scalar diffraction theory (ignoring polarization effects), the complex-valued electric field at the image is given by the Fraunhoffer integral over S, which is just a Fourier transform:

$$E(\xi, \zeta) = \frac{iE_0}{\lambda f} \iint_S e^{-\frac{2\pi i}{\lambda f}(x\xi + y\zeta)} A(x, y) dx dy \tag{1}$$

where (ξ, ζ) are coordinates in the image. The point spread function (PSF) is then defined as the magnitude squared of the image plane field:

$$P(\xi, \zeta) = \frac{E_0^2}{\lambda^2 f^2} \left| \iint_S e^{-\frac{2\pi i}{\lambda f}(x\xi + y\zeta)} A(x, y) dx dy \right|^2 . \tag{2}$$

The high contrast imaging problem is to find an apodization function $A(x, y)$ that produces a point spread function, $P(\xi, \zeta)$, with the property that certain regions of the image plane, called dark holes, have a specified high contrast relative to the peak at $P(0, 0)$. Since $A(x, y)$ can only attenuate, it is a property of all apodized pupil coronagraphs that the throughput of the system is reduced. We can thus formulate the problem as an optimization that chooses an A with maximum throughput under a constraint of high contrast in the largest possible region of the image with a given inner working angle. As described in previous chapters, the *inner working angle* of a coronagraph is the smallest angle on the sky at which high contrast is achieved. For the apodized pupil coronagraphs described here, it is the location in the image plane closest to the center where the PSF still exhibits has high-contrast.

Note: if there is room, we could add here the nondimensionalization of the PSF equation and perhaps provide specific equations for various metrics such as throughput, airy throughput, pseudo-area, etc.

Because we will be looking at various geometries, it is useful to find the form of the Fraunhoffer integral for a few special cases. The first is a square aperture where we assume that the apodization function is separable as the tensor product of two one-dimensional functions, $A(x, y) = A_x(x)A_y(y)$. If in addition the two factors are even functions (i.e., $A_x(-x) = A_x(x)$ and $A_y(-y) = A_y(y)$), then the Fraunhoffer integral in Eq. 1 becomes:

$$E(\xi, \zeta) = \frac{E_0}{\lambda f} \int_0^{D/2} A_x(x) \cos(2\pi\xi x) dx \int_0^{D/2} A_y(y) \cos(2\pi\zeta y) dy. \tag{3}$$

A special case of the separable apodization is the purely one-dimensional apodization, $A(x, y) \equiv A(x)$. In that case, the field in Eq. 3 simplifies to

$$E(\xi, \zeta) = \frac{iE_0}{\lambda f} \frac{2\sin(\pi\zeta)}{\pi\zeta} \int_0^{D/2} A(x) \cos(2\pi\xi x) dx. \tag{4}$$

For a circular aperture we can change to polar coordinates (r, θ) in the pupil. If we similarly assume an apodization that is only a function of r, then the Fraunhoffer integral can be completed around the azimuthal direction, θ, resulting in the well-known *Hankel transform*:

$$E(\rho) = \frac{2\pi i}{\lambda f} \int_0^{D/2} A(r) J_0 \left(\frac{2\pi r \rho}{\lambda f} \right) r dr \tag{5}$$

where J_0 is the zero order Bessel function of the first kind and $E(\rho)$ is also circularly symmetric with coordinate $\rho = \sqrt{\xi^2 + \zeta^2}$ in the image plane.

2. Apodized Pupil Coronagraphs

In this section, we describe smooth apodization functions that can be applied to a telescope to provide high contrast. The simplest scenario arises if the telescope has a clear aperture—no central obstruction and no spiders. In this case, it makes sense to look for an azimuthally symmetric apodization defined by a function $A(r)$. There are various ways to approach the design problem. One is to try to optimize the light throughput subject to the constraint that the relative intensity in the image plane is below some threshold contrast level in some annulus around the center of the point spread function:

3. Shaped Pupil Coronagraphs

4. Shaped Pupil Lyot Coronagraphs

5. Final Remarks