# Measuring the Astronomical Unit from Your Backyard

Two astronomers, using amateur equipment, determined the scale of the solar system to better than 1%. So can you. By Robert J. Vanderbei and Ruslan Belikov

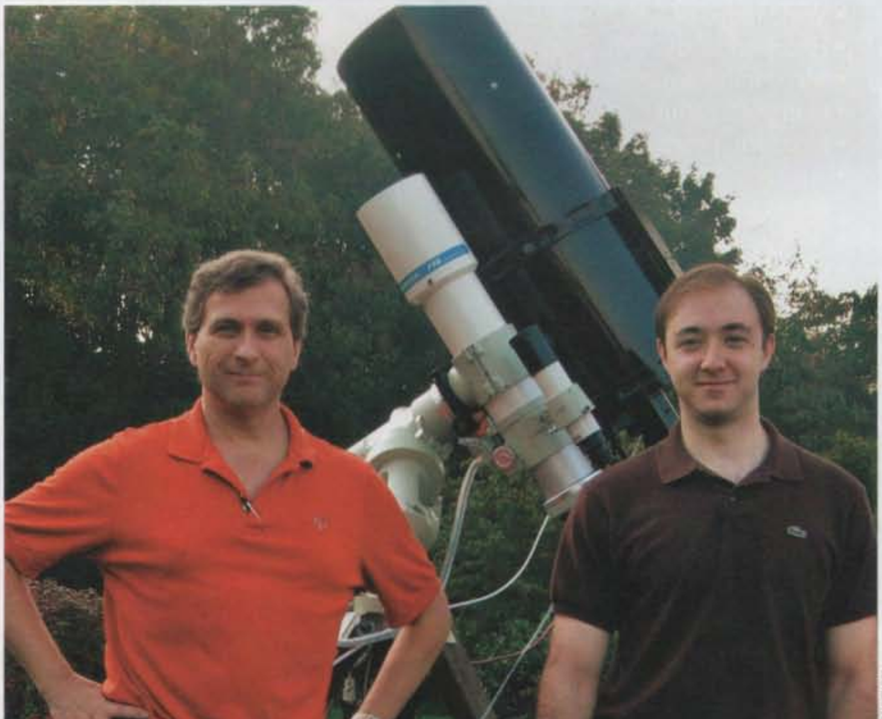
HERE ON EARTH we measure distances in millimeters and inches, kilometers and miles. In the wider solar system, a more natural standard unit is the *astronomical unit:* the mean distance from Earth to the Sun. The astronomical unit (a.u.) equals 149,597,870.691 kilometers plus or minus just 30 meters, or 92,955,807.267 international miles plus or minus 100 feet, measuring from the Sun's center to Earth's center. We have learned the a.u. so extraordinarily well by tracking spacecraft via radio as they traverse the solar system, and by bouncing radar signals off solar-system bodies from Earth. But we used to know it much more poorly.

This was a serious problem for many branches of astronomy; the uncertain length of the astronomical unit led to uncertainty in every distance beyond Earth. Here is what Fred Whipple said about this in his 1941 book *Earth, Moon and Planets:*

> By measuring the positions of the planets or asteroids and by applying Newton's law, astronomers can calculate planetary orbits about the Sun and predict future positions with high accuracy. The relative distances can be calculated with a precision equal to that of the most precise distances measured on the Earth, to about one part in a million. The surprising difficulty lies in the fact that these accurate distances are all in terms of the astronomical unit, the mean distance from the Earth to the Sun, not in terms of feet or miles. For purposes of prediction this uncertainty makes almost no difference, but no scientist relishes the use of a measuring rod with an unknown length.

Uncertain distances to everything in the solar system meant uncertainty in their sizes, densities, and other characteristics, and in the mass of the Sun. Moreover, the significance of the a.u. extended beyond the solar system. Distances to nearby stars are based on stellar parallax, which depends on the diameter of Earth's orbit. And most of the larger distance measures throughout the cosmos are based in some way on distances to nearby stars. Determining the astronomical unit was as central an issue for astronomy in the 18th and 19th centuries as determining the Hubble constant — a measure of the universe's expansion rate — was in the 20th.

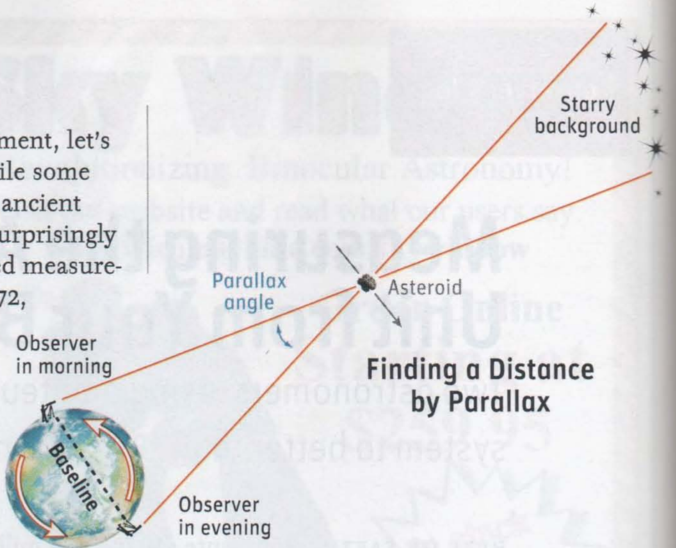
Astronomers of a century and more ago devised various ingenious methods for determining the a.u. In this article we'll describe a way to do it from your backyard — or more precisely, from any place with a fairly unobstructed view toward the east and west horizons — using only amateur equipment. The method repeats a historic experiment performed by Scottish astronomer David Gill in the late 19th century. Although not the first, it was the most accurate measurement in its day.



MARISA VANDERBEI

Princeton University scientists Robert Vanderbei (left) and Ruslan Belikov pose with the setup they used to measure the astronomical unit. About six years ago Vanderbei rejoined the world of amateur astronomy, buying a 3.5-inch Questar, then a 4-inch Takahashi refractor, and most recently the 10-inch RC Optical Systems telescope shown here. Belikov is also an avid amateur astronomer and can frequently be found tinkering with his new 10-inch Meade LX200R.

### Narrowing In, Century by Century



**Finding a Distance by Parallax**

Before describing Gill's experiment, let's review some more history. While some speculate that Eratosthenes in ancient Greece may have arrived at a surprisingly good value, the first undisputed measurement of the a.u. dates from 1672, when Jean Richer and Gian Domenico Cassini measured the *topocentric parallax* of Mars — the difference in its positions on the sky seen from their changing positions as Earth rotated. From this they deduced a value of 140 million km for the a.u. Not bad for 1672; they were off by only 7%.

That value held for the next 100 years. Early in the 18th century Edmond Halley devised another method based on timing transits of Venus across the face of the Sun, something that occurs less than twice per century (*S&T*: February 2004, page 46, and May 2004, page 32). The idea was to time the moment when the edge of Venus crossed the edge of the Sun as seen from sites spread as widely as possible across Earth. The transits in 1761 and 1769 yielded a value of about 153 million kilometers. Although this value is more nearly correct than Richer and Cassini's, no one knew it at the time; the measurement uncertainty was rather large, mainly because of difficulties in marking the contact times precisely due to the so-called black-drop effect in the planet's silhouette on the Sun. The next Venus transits took place in 1874 and 1882. Again, and for the same reason, the results were somewhat disappointing.

Owing to this problem with Venus transits, in 1877 David Gill traveled to Ascension Island in the South Atlantic to observe Mars during its opposition that year. He measured the Red Planet's position against the stars when seen low in the east just after sunset, and low in the west just before sunrise, repeatedly for several weeks. He was able to determine the daily wobble in Mars's apparent position caused by his own changing position as Earth carried him along with its rotation through the night. That is, he determined a parallax angle for Mars relative to a baseline that was a known and large fraction of Earth's diameter. Given the parallax angle and the baseline, simple

## Asteroid Targets

For readers who would like to try our experiment this season, listed below are some asteroids that have close oppositions in the coming months. The coordinates are for midnight (0:00) Eastern Standard Time (5:00 Universal Time) on the evenings preceding the dates given, but each object is nearly as close for a few weeks before and after. Use a planetarium program or an ephemeris for other dates.

### Selected Asteroids at Their Opposition Dates, 2006–07

| Date ($5^h$ UT) | Object | RA | Dec. | Dist. (a.u.) | Mag. |
|---|---|---|---|---|---|
| Dec. 2 | 7 Iris | $3^h 00.7^m$ | +21° 11′ | 0.8910 | 7.3 |
| Dec. 16 | 888 Parysatis | $4^h 41.0^m$ | +3° 32′ | 1.2538 | 12.4 |
| Jan. 2 | 44 Nysa | $6^h 28.2^m$ | +19° 28′ | 1.0926 | 9.0 |
| Jan. 16 | 144 Vibilia | $6^h 29.1^m$ | +26° 47′ | 1.5960 | 11.5 |
| Feb. 2 | 37 Fides | $7^h 45.5^m$ | +26° 05′ | 1.3541 | 10.3 |
| Feb. 16 | 20 Massalia | $8^h 29.9^m$ | +17° 44′ | 1.1582 | 9.0 |
| Mar. 2 | 116 Sirona | $9^h 36.2^m$ | +20° 30′ | 1.4453 | 11.1 |

trigonometry allowed Gill to compute the distance from Earth to Mars during that opposition — and from that, the scale of the solar system and the length of 1 a.u. Gill's value was remarkably accurate — within 0.2% of the correct value!

The asteroid 433 Eros prompted the next historic attempt. This small object was discovered in 1898, and astronomers quickly realized that it comes much closer to Earth than Mars does. And since it appears as a point of light rather than a disk, its position against the stars would be easier to measure. In 1900 an international observing campaign yielded a result that was better than Gill's and remained the textbook value for years. Eros was ranged by radar for the first time in 1975, opening the modern era of extraordinary precision.

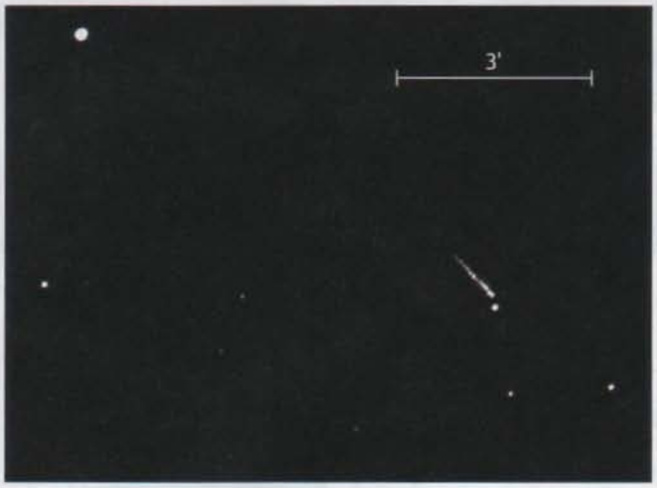
### A Backyard Project

Today's amateur equipment makes it fairly easy for anyone to measure the a.u. All you need are reasonably good views of the east and west horizons and a quality telescope and CCD camera. Close passes of Eros come infrequently, but there are lots of other asteroids out there. On almost any night at least one is having a relatively close opposition, making it a good target.

We measured the a.u. last summer from a backyard in suburban New Jersey. On the afternoon of August 8th, we looked at our planetarium program (*Cartes du Ciel*, but any such program will do) to see what asteroids were near opposition. We picked 474 Prudentia because it was particularly close to Earth — according to *Cartes du Ciel*, it was only 0.9309 a.u. distant at the time. We confirmed this value by a visit to the invaluable Horizons solar-system data and ephemeris computation service, offered by the Jet Propulsion Laboratory at http://ssd.jpl.nasa.gov/?horizons.

That evening we set up our telescope in the yard. After polar-aligning it we used the scope's Go To system to point to Prudentia. We had to wait 20 or 30 minutes for the asteroid to clear the trees. Then we took 10-second exposures every 10 minutes for about 90 minutes. Next, one of us set the alarm clock for 3:30 a.m. and returned to take 10-second exposures every 5 minutes (just to be sure) for about an hour prior to dawn. The next evening we repeated the procedure, collecting a third set of CCD images.

An asteroid moves a fair amount in both right ascension (RA) and declination



The authors used the 10-inch telescope pictured on page 91 and a Starlight Xpress SXV-H9 CCD camera to capture this streak of the 12th-magnitude asteroid 474 Prudentia. It's a stack of 12 unfiltered 10-second exposures taken at 5-minute intervals on August 9th Universal Time.

in 24 hours: the total angular displacement for Prudentia was about 15 arcminutes. Superimposed on this essentially uniform, linear motion is an apparent oscillating motion caused by our backyard swinging around Earth's axis once a day. Since Earth rotates about a fixed polar axis, we only need to measure the asteroid's oscillation in RA.

We used *MaxIm DL*'s astrometry tool to determine the asteroid's RA and declination relative to the stars in each of our images. The diagram below shows a raw plot of RA versus time. Clearly, it is dominated by the asteroid's uniform linear motion. It's easy to estimate this motion just by comparing two images 24 hours apart. We can then subtract it out to see Prudentia's *residual motion*, as shown in the diagram at lower right. We know that the residual motion has to be a nearly-24-hour sine curve. Shown in the diagram is the curve of this type that best fits our data.

Our observing site was at latitude 40° 27′ north. Simple trigonometry tells us that the baseline for this latitude is 9,693 km (the diameter of Earth is 12,738 km, which we multiply by the cosine of our latitude to get the baseline). From the diagram below, it's clear that the peak-to-peak RA oscillation ($\theta$) was about 13 arcseconds. To determine $\theta$ as accurately as we could, we did a regression analysis (using a quadratic rather than a simple linear equation for the background motion), which gave a peak-to-peak value of 14.47 arcseconds.

Given the known baseline ($b$) in kilometers and the angle, we could compute the distance ($d$) in kilometers to Prudentia using the formula $d = b/2\sin(\theta/2)$, which gives 138,200,000 km. Therefore 1 a.u. equals 138,200,000 km divided by 0.9309 a.u. (the ephemeris distance that we looked up), or 148,500,000 km. That's only 0.7% below the correct value!

In just 26 hours and armed only with hobby equipment, we measured the a.u. almost as well as David Gill did — with a small fraction of his skill and effort. The result could surely be improved by conducting many observations over many nights, and from a site closer to the equator (giving a longer baseline), and in better seeing. But it's amazing to realize how easy it is these days to measure a fundamental quantity that many ancient and medieval astronomers would have gladly given their lives to learn.

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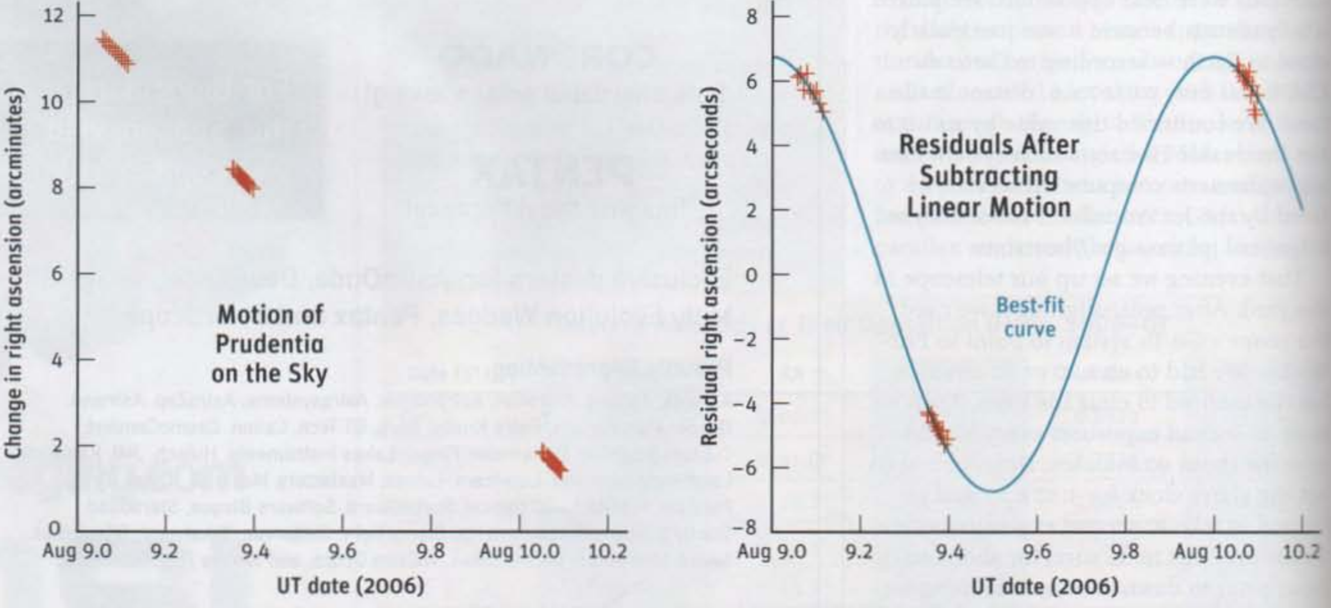

*Left:* This diagram shows the changing right ascension (RA) of Prudentia as it moved across the sky. Its motion with respect to the background stars was nearly linear. The authors took clusters of measurements (red crosses) on the evening of August 9, 2006 UT, the next morning, and the next evening. *Right:* After uniform linear motion was subtracted out, what remained were the residuals: the apparent sinusoidal oscillations of Prudentia in right ascension due to the observers' changing position as Earth turned.