Measuring the Astronomical Unit

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ABSTRACT

Subject headings: Kepler's law, Astronomical Unit, asteroid, parallx

The distance from the Sun to the Earth is called the *astronomical unit* (AU), and has the currently accepted value of $149,597,870,691 \pm 30$ meters. In earlier centuries measuring this distance was a fundamental problem in astronomy. There is some evidence that as early as in ancient Greece, Eratosthenes may have arrived at a very accurate value of 149 million km. The first undisputed successful measurement, however, would have to wait until 1672, when Jean Richer and Giovanni Domenico Cassini measured the parallax of Mars at two different locations on Earth, and from it deduced a value of 140 million km for the AU. That value would hold for the next hundred years. In the 18th and 19th centuries, Edmund Halley devised a popular method for measuring 1 AU, based on timing the rare transits of Venus, which occur less than twice per century. (This subject was extensively covered in this magazine in honor of the much awaited 2004 transit. See, e.g. the February and May 2004 issues, pp. 46-54 and 32-37, respectively.) The transits of 1761 and 1769 yielded a value of about 153 million kilometers. Although more accurate than Richer and Cassini's result, this transit estimate was still rather poor, mainly because of difficulties in accurately measuring the contact times due to the fact that the Sun dims at its edge (limb darkening) and because the Venus image has a diffuse edge. Together, these effects created what is called the *black-drop effect* which dramatically compromised the measurement. The next Venus transits were in 1874 and 1882. Again, and for the same reasons, the results were somewhat disappointing.

Because of these difficulties with Venus transits, in 1877 a Scottish astronomer by the name of David Gill traveled to Ascension Island in South Africa to observe Mars during its opposition of that year (see article by J. Donald Fernie, American Scientist, July-August 2006, pp.308-310). By observing Mars just after sunset and just before sunrise over the course of at least two days, one can use the diameter of Earth as a baseline in a parallax determination for Mars. Then, by knowing the diameter of Earth and the latitude of the observer, it is easy to compute the baseline for the parallax measurement. Given the parallax and and the baseline, simple trigonometry allowed David Gill to compute the distance from the Earth to Mars during that opposition. Finally, it was well understood in 1877 that Kepler's law tells us the distances from the Sun to Mars (and to all other bodies in the solar system) in terms of AU. Thus, given the distance between Earth and Mars in absolute units, say miles or kilometers, it is simple to figure out what 1AU is. It turns out that David Gills' measurements proved to be very accurate – within 0.2%.

Given a location with a reasonably good view of the eastern and western horizons, it is fairly easy to repeat David Gills' experiment. But, to do exactly what he did requires one to wait for a Mars opposition and these come along only every couple of years. For those of us who are impatient, a very similar experiment can be done using an asteroid at opposition.

We decided to do this experiment a few months ago. But we needed to wait until the weather forecast indicated a stretch of clear skies that would last for at least 30 hours. It took a few months before we came upon a pair of clear nights that was also good for our work schedules. On August 8 and 9 we got our chance. In the afternoon of August 8, we looked at our planetarium program (specifically, Cartes du Ciel but any such program would do) to check out which asteroids were near opposition on this night. There were several. We picked (474) Prudentia because it is particularly closeby. According to Cartes du Ciel (and confirmed by a visit to JPL's Horizons ephemeris system), this asteroid was only 0.9309 AU from Earth at the time of our measurements. You the reader might object by saying that Cartes du Ciel probably knew the distance in kilometers and converted it to AUs using exactly the number we are trying to compute. But, it is unlikely that the data comes that way since, because of Kepler's law, the AU is the more fundamental unit for the purposes of astronomical measurement. Nonetheless, anyone who is worried that this is cheating could simply use the planetarium program to figure out the number of years and days from one opposition of the asteroid to the next. From that it is possible to compute the distance from the Sun in AUs and simple but tedious geometry then would allow you to compute the Earth-asteroid distance in AUs.

In the evening of August 8, we set up our telescope equipped with a CCD camera. We polar aligned the equitorially mounted telescope and used the mount's goto system to point to Prudentia. We had to wait about 20 or 30 minutes for the asteroid to rise above the trees. Once it was clear of the trees, we took one 10 second image every 10 minutes for about 90 minutes. One of us set his alarm for 3:30am and went out and took a second set of measurements, this time using 10 second exposures every 5 minutes, for about an hour prior to dawn. Then the next evening we collected another set of data points.

Because both the Earth and the asteroid are moving around the Sun, the asteroid moves a fair amount in both RA and Dec over the course of 24 hours. The total displacement for Prudentia was about 15 arcminutes. But, if one subtracts this essentially uniform motion, what remains is an oscillatory motion caused by the fact that the Earth is rotating on its axis once a day. Since the Earth's rotation is a turning in the RA axis, the biggest oscillatory effect is in RA. In fact, if Prudentia had been exactly on the equator (Dec=0), then there would be no oscillation in Dec. In fact, Prudentia was at Dec = -06 degrees. Hence, the Dec oscillation was very small. So, we concentrate our analysis on the RA coordinate.

We used MaximDL's astrometry tool to extract the asteroid's RA and Dec from each of our images. Figure 1 shows a raw plot of RA vs. time. Clearly, it is dominated by the uniform motion. But, it is easy to estimate the uniform motion simply by comparing two images taken as closely as possible to 24 hours apart. Once the uniform motion is known, we can subtract it to see the residual motion. Figure 2 shous the residual motion that we found for Prudentia. From this sinusoidal motion, it is easy to estimate (or compute via regression) the total peak-to-peak displacement in RA. Our observing location is at a latitude of $40 \deg 27'$. Simple trigonometry then tells us that the baseline for this latitude is 4847 km (the radius of the Earth at the latitude is 6369 km, which we multiply by the cosine of the latitude to get the baseline). From Figure 2 it is clear that the peak-to-peak RA oscillation is about 13 arcseconds. A regression analysis gives 13.49 arcseconds. Given the baseline and the angle, we compute the distance

$$\begin{aligned} d_{\text{Earth-Prudentia}} &= d_{\text{baseline}}/(2\sin(\theta/2)) \\ &= 148,217,000\text{km}. \end{aligned}$$

This distance is 0.9309AU and so

$$\begin{aligned} 1\text{AU} &= 148,217,000/0.9309 \\ &= 159,219,000\text{km}. \end{aligned}$$

It turns out that Prudentia might not have been an ideal choice as it has a high eccentricity (0.2) and inclination 9°. These two factors are the main source of error in our computation above since the retrograde motion is not really linear as we assumed. In fact, when we plotted the retrograde motion in our planetarium program we saw that it is a rather pronounce S-shaped curve. So, we enhanced our regression analysis replacing the assumption of linearity with an assumption the the retrograde motion is well approximated by a parabola. With this one simple change, we get

$$1\text{AU} = 147,947,000\text{km},$$

which is only 1.1% below the correct value.

Thus, within the span of 26 hours, equipped only with amateur equipment, we have measured 1AU with about a 1% error, which is about as accurate as David Gill's estimate more than 100 years ago. And, we only spent a small fraction of the effort that David Gill did. Conducting many observations over many days from a location that is closer to the equator with good weather and seeing will undoubtedly improve the results. But, it is amazing to think that these days you can go into your backyard and in 24 hours measure a quantity that many ancient Greek and medieval astronomers would have gladly given their lives to know.

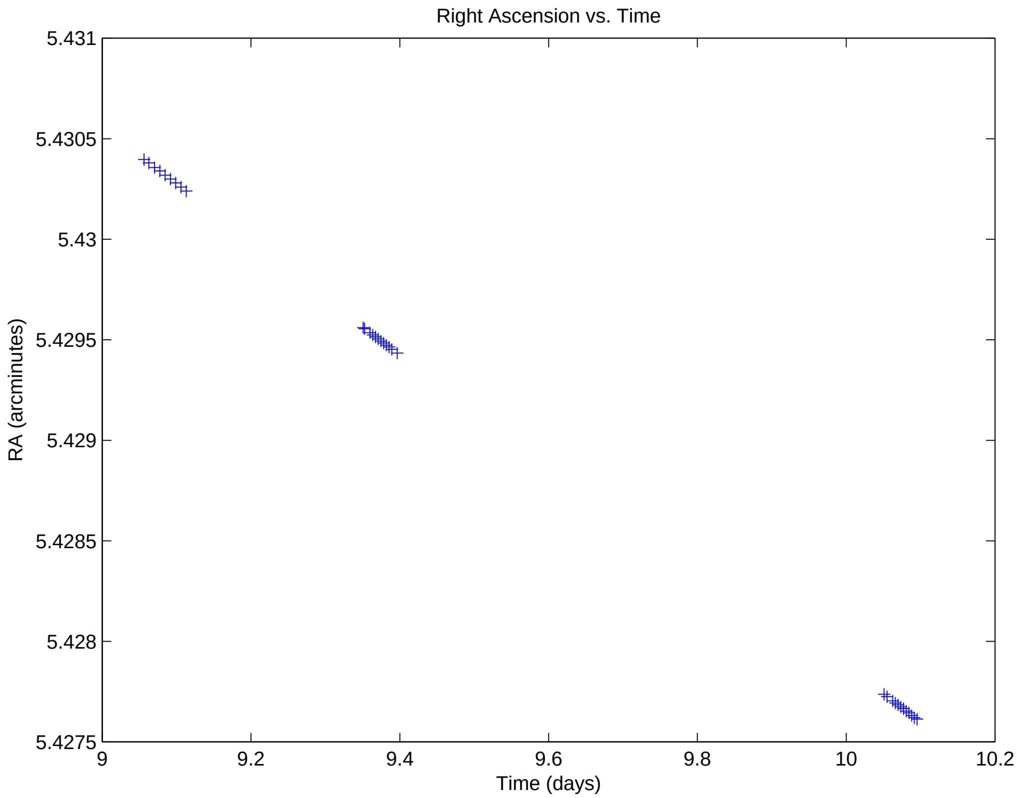

Fig. 1.— Absolute RA position of Prudentia as it moves across the sky. The motion is mostly linear and retrograde with respect to the stars, as earth overtakes the asteroid. Clusters of measurements taken during the evening, morning, and the following evening can clearly be seen.

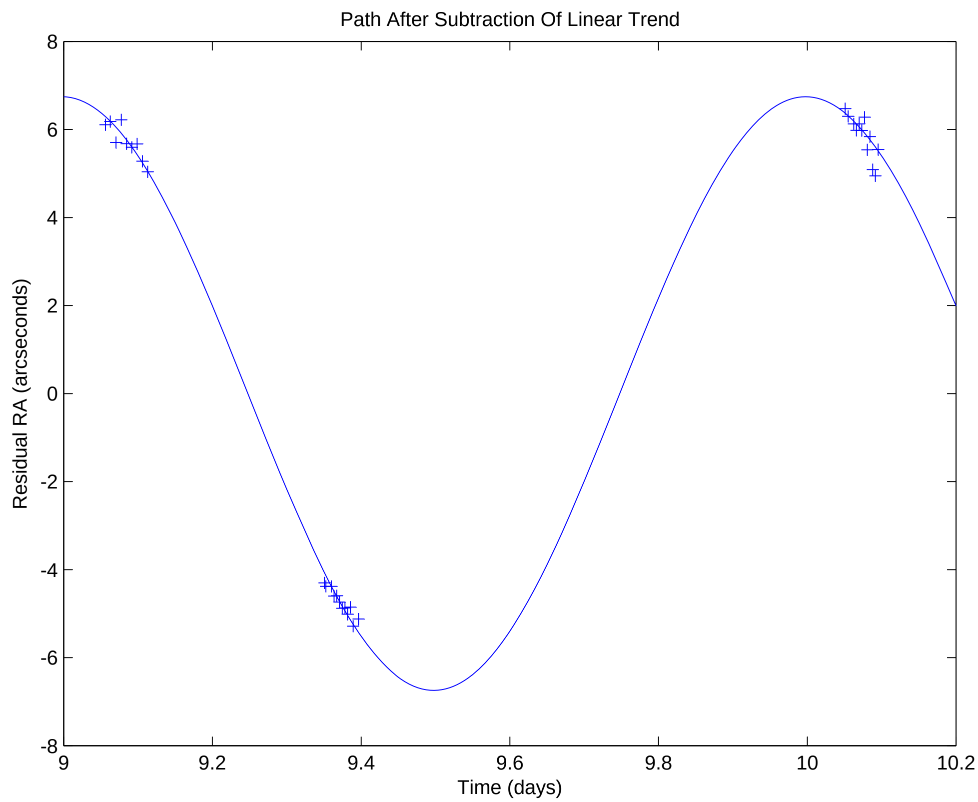

Fig. 2.— After the uniform linear motion due to the Earth's and Prudentia's solar orbits is subtracted out, the residual shows the sinusoidal RA motion of Prudentia across the sky due solely to the rotation of the Earth.