

The Retrograde Motion of Mars with a Sextant

Bob Cava, Bob Vanderbei, and Liz Seibel, Princeton NJ

This project started when one of us (Bob C.) came across a statue of Jan Hevelius in Gdansk Poland. (Figure 1). Hevelius mapped the moon and stars using naked eye observations 350 years ago, so why not try it now? Monster angle-measuring instruments like Hevelius used are not things that you see for sale nowadays, but the modern equivalent is a nautical sextant. Professional quality real nautical sextants are too expensive for just fooling around, and so a plastic one and two books about how to use it (“How to use plastic sextants” by David Burch and “The Sextant Handbook” by Bruce Bauer) were what to get to try this kind of thing. During a trip to Florida in early 2016, Bob and his brother-in-law measured high noon with the plastic sextant as a test and got their position on earth right to within about 3 miles, showing that it is possible for novices to use a sextant reasonably well. A little later, this same brother-in-law found a used, 1970s vintage professional metal sextant at a garage sale that cost less than the new plastic sextant did in the first place and sent it up to New Jersey.

So with sextants in hand (Figure 2) it was time to do some measuring. The plastic sextant is much flakier optically and mechanically than the real sextant, but it’s lighter and so was much less tiring to use during the measurements, which were often time-consuming. Also, the second-hand real sextant has smaller mirrors than the plastic one, making fainter stars harder to see. For calibration purposes, Bob measured the distances between ten pairs of fixed stars with separations between about 3 degrees and 25 degrees with both sextants and compared his results to the known angular separations - the plastic sextant got the angles right to (RMS errors) within 0.05 degrees on average and the real sextant got the angles right to within 0.02 degrees on average, in neither case with any measurement outliers. The measurements seemed good, and, in the end, the increased accuracy of the real sextant generally wasn’t worth the arm fatigue and eye strain for what had to be done, so Bob mostly stuck with the lightweight, plastic sextant; the real sextant served periodically as a sanity check.

The original idea was to mount the sextant on a lightweight tripod with a photo camera ball head to eliminate fatigue and to steady things up, but because the measurements for the project involved defining the plane of measurement every time for three points – observing eye, Mars, and reference star - it turned out to be much easier to just find the orientation of the measurement plane by hand. And since sextants are good for measuring angles on rocking ships, just hand-holding one was the best way to get good measurements anyway – “steadying things up” was not necessary and would have probably been worse due to the rocking technique needed to make the measurements

Mars seemed like a good choice for following with the sextants because its proper motion against the fixed stars would be large, and adding to the fun would be that around opposition it would exhibit retrograde motion. Observations involved measuring the angular separations between Mars and three to seven reference stars, and often Saturn (which moves too), and sometimes involved playing peekaboo with clouds that covered some of the target objects for long periods of time in the middle of the measurements. Antares and beta and delta Scorpio, zeta Ophiuchus and beta Libra were used as reference stars on many nights; and sigma and tao Scorpio and alpha Libra were used whenever they were visible in spite of the light pollution and general haze in Princeton. Measurements were made of Mars’ position on 68 nights over a period of 7 months to collect the data. Luckily, it was possible to crawl out of bed in the wee morning hours to take the early measurements from inside a sunroom, (Its cold in New Jersey in February!) but some of the measurements were more challenging – on business trips to

various places, and on a cruise ship at odd hours, for example. Eventually the drama began to unfold during more normal observing hours, finally ending around dusk in the fall, when some of the faint stars were tough to see in the twilight and mosquitos had become a factor.

It felt good to finish the observing project, and that could have been the end of it. But then Bob realized that the data could probably be analyzed quantitatively – that given the dates and the positions of Mars, someone (but not him) could probably extract quantitative information about the orbit of Mars from the observations. This is where Liz and Bob V came in. Liz did the first round of data plotting and analysis (See Figure 3 for example), and then Bob V. analyzed the data to determine Mars' orbit through a fit to an orbital model.

As input, Bob took the dates of the observations and the angular separations to the reference stars, and then determined Mar's position in ecliptic coordinates for all 68 observing sessions using a linear least-squares algorithm. (See Figure 4.) He then fed the dates and positions of Mars into a highly nonlinear least-squares regression model where he assumed that Mars travels in a circular orbit inclined to the plane of the earth's orbit. The fitting parameters were the relative radii of the orbits of the earth and Mars, the angle of Mar's orbital plane with that of earth (aka the ecliptic latitude), and the ecliptic longitude of the ascending node. The resulting least-squares-fitted orbit and the observations are shown in Figure 5. The numbers Bob got for Mars' orbital parameters are: Mars' orbit Semimajor axis = 1.530 au, Inclination = 2.1 degrees, and Longitude of Ascending Node = 48.5 degrees. Compare these to the known values: Semimajor axis = 1.524 au, Inclination = 1.85 degrees, Longitude of Ascending Node = 49.56 degrees and they look pretty good, right? Too bad we were a few hundred years too late to be first to figure it out. Notice by comparing the observed and calculated positions on a particular date (see the asterisks in Figure 5) that the model fit is not perfect. We think that the observations are over a long enough period, 7 months, that the ellipticity of Mars' orbit should likely be taken into account – but fitting the data using Kepler's law makes things more complicated and we haven't tried it yet.

This was a fun project, not in the least because it motivated one of us to go out to observe on many nice nights when he might have been asleep or doing something boring instead of watching something cool happening in the sky. Also, analyzing the data together was lots of fun. The data clearly show that Mars really does move a lot and even goes backwards quite a bit with respect to the fixed stars during an opposition - something one doesn't always pay attention to. Finally, what could more fun than fooling around with unfamiliar equipment like sextants, taking some measurements on beautiful nights, and then fitting the data to determine a planetary orbit? At least one of us (Bob C.) can't think of anything that would be better. By following Mars with a sextant for seven months, and then figuring out its orbit by fitting the data, we sure gained a lot of respect for the ingenuity and grit of early astronomers like Hevelius.



Figure 1 The inspiration for the project. One of the authors (Bob C.) in Gdansk Poland with a statue of Jan Hevelius.



Figure 2 The two sextants used to make the measurements. The plastic sextant is on the left and the “real” one, a second-hand professional sextant from the 1970s, is on the right.

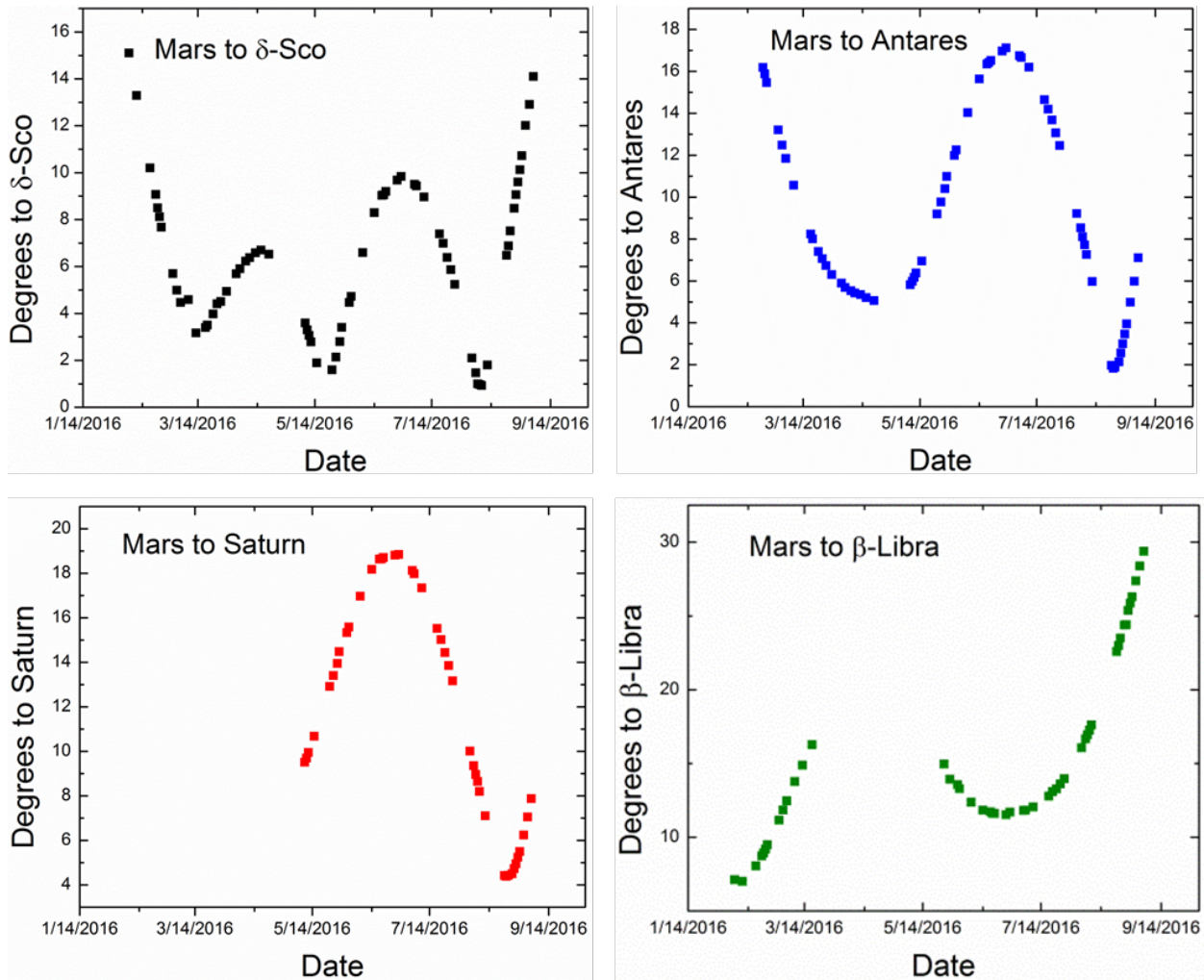


Figure 3. The first steps in the data analysis. Measured separations of Mars (in degrees) to some of the nearby fixed stars (and Saturn), plotted versus local calendar date. Annoying clouds, haze, the moon, out of town trips, and light pollution created some gaps in the data, and the separation to Saturn wasn't measured until about halfway through the project.

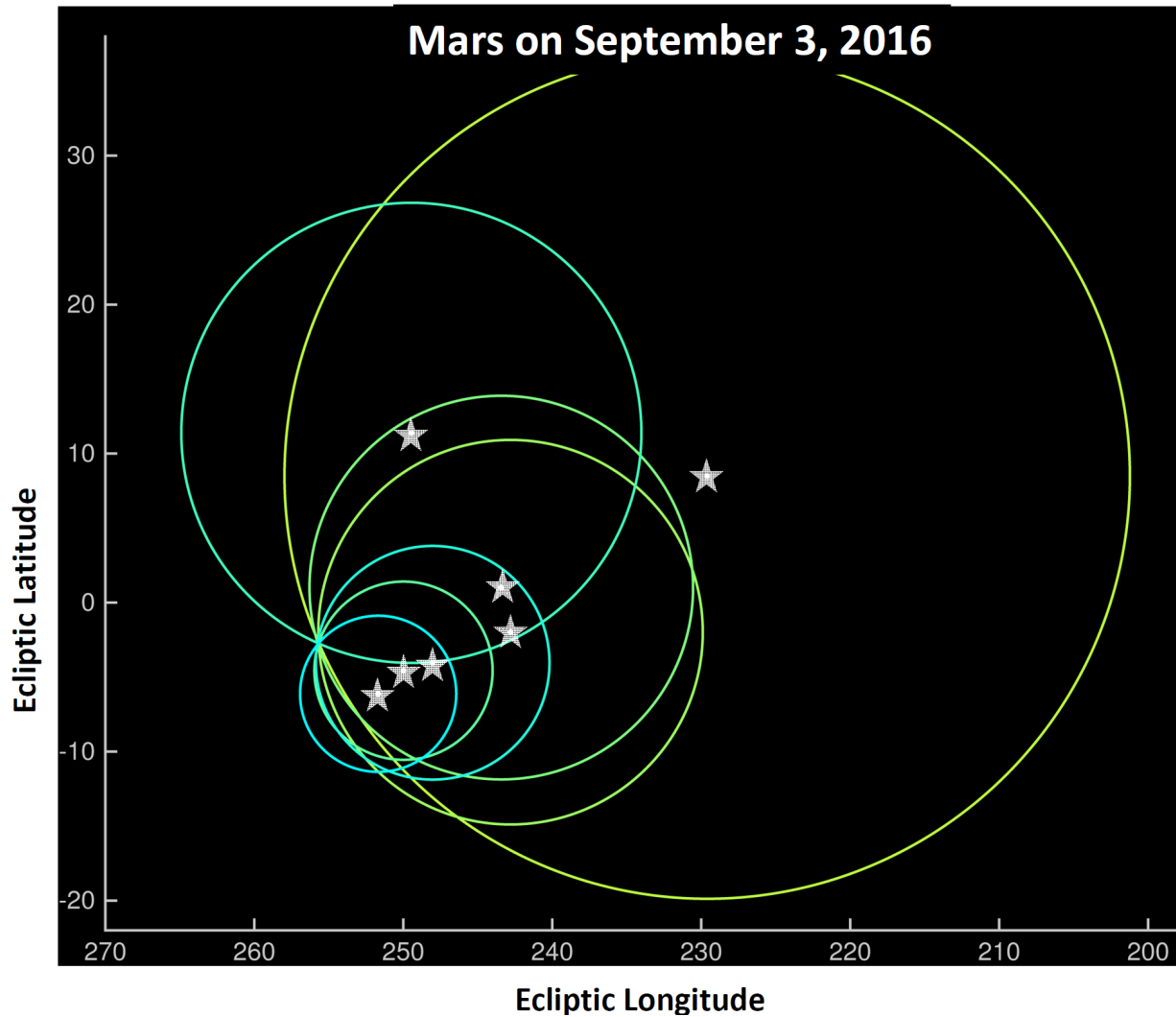


Figure 4 How the position of Mars was determined. The circles show the sextant-measured angular separations of Mars from 7 reference stars (September 3 2016 had a nice clear night.) Mars is at the intersection of all the circles, and the reference stars in Scorpio, Ophiuchus and Libra are shown as star symbols. Longitude shown on the horizontal axis, altitude on the vertical axis, in ecliptic coordinates.

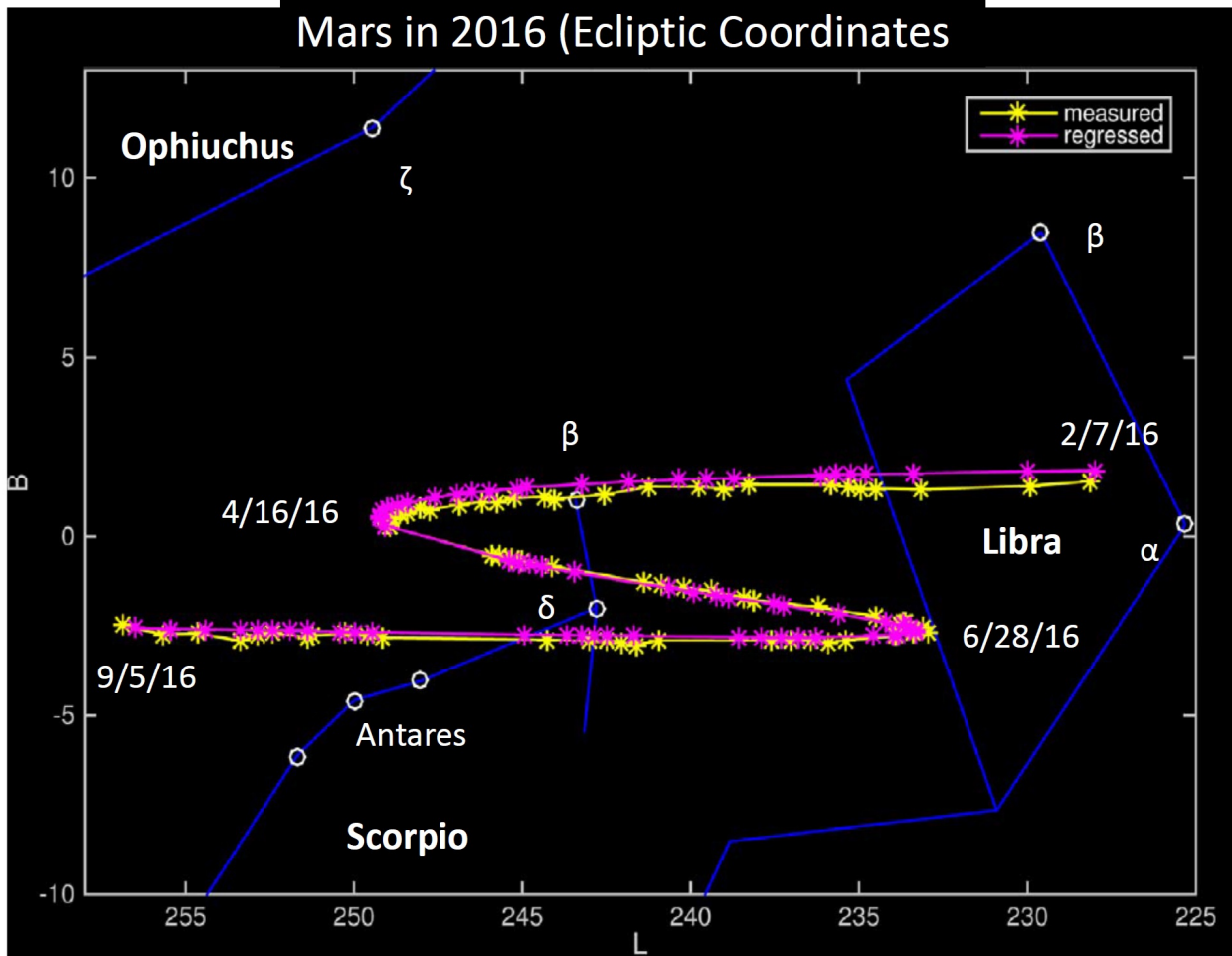


Figure 5 The retrograde motion of Mars with a sextant. 2016 apparition. Observations: yellow points and line, fitted data: purple points and line. Mars started at the upper right of this path in February and departed on the lower left of the path in September. The retrograde motion, resulting in an S-shaped path, is clearly seen – opposition is in the middle of the retrograde segment. Parts of the constellations in the area (Scorpio, Libra and Ophiuchus.) are shown by blue lines, with the stars as open circles. The fit was used to determine the orbit of Mars.