An external flower-shaped occulter flying in formation with a space telescope can theoretically provide sufficient starlight suppression to enable direct imaging of an Earth-like planet. Occulter shapes are scaled to enable experimental validation of their performance at laboratory dimensions. Previous experimental results have shown promising performance but have not realized the full theoretical potential of occulter designs. Here, we develop a two-dimensional diffraction model for optical propagations for occulters incorporating experimental errors. We perform a sensitivity analysis, and comparison with experimental results from a scaled-occulter testbed validates the optical model to the $10^{-10}$ contrast level.

The manufacturing accuracy along the edge of the occulter shape is identified as the limiting factor to achieving the theoretical potential of the occulter design. This hypothesis is experimentally validated using a second occulter mask manufactured with increased edge feature accuracy and resulting in a measured contrast level approaching the $10^{-12}$ level – a better than one order of magnitude improvement in performance.

© 2016 Optical Society of America

1. INTRODUCTION

The direct imaging of an Earth-like planet around a nearby stellar system, or an exo-Earth, is one of the most important scientific goals of astronomy but requires an extraordinary capability for high-contrast imaging. The contrast ratio between the peak of the stellar point spread function (PSF) and that of a modeled exo-Earth in visible and near-IR bands is ten orders of magnitude [1]. Thus, the fainter exo-Earth is located at a small angular separation from its host-star even for the nearest planetary systems to our Sun and would be indistinguishable from the diffraction wings of the stellar PSF.

An exo-Earth high-contrast imaging capability will require the usage of a starlight suppression system. The external occulter, sometimes referred to as a starshade, was first proposed by Lyman Spitzer in 1962 [2] and is an alternative to an internal coronagraph that does not require an active wavefront control system. The occulter spacecraft is flown in formation with a space-telescope along its line-of-sight to the host star, and blocks most of the starlight from entering the telescope pupil. The residual starlight due to diffraction must be controlled by optimizing the shape of the occulter. Occulter designs proposed for space missions such as EXO-S, THEIA, and NWO [3–5] use binary occulters – fully transmissive or opaque – which approximate an apodization function through N-fold circular symmetry by introducing a set of petals [6]. These designs are typically tens of meters in diameter and operate at tens of thousands of kilometers separation from the space telescope.

A numerical optimization process has been developed for the design of the occulter apodization function [7]. This and other design approaches are based on scalar diffraction theory. Full-scale petals have been built and assembled; their individual edge shape and assembled configuration has been precisely and repeatedly measured to predict their optical performance based on existing manufacturing capabilities [8]. Nonetheless, the performance prediction is also based on scalar diffraction modeling. Because a full-scale occulter cannot be tested on the ground, it is therefore important to experimentally verify the performance of scaled occulters in the laboratory to validate the optical models used for their design and performance prediction at the contrast level required for the space mission.

Previous occulter experiments have demonstrated high-contrast but nonetheless not to the full potential of the scaled occulter design. Some of these earlier laboratory tests have been limited by phase aberrations due to surface errors from colli-
mating optics, diffraction effects due to the finite extent of the optics and laboratory enclosure or field site, and scattering from the support structure used to suspend the occulter mask [9–11]. At Princeton, we have developed a proof-of-concept testbed (described in detail in Section 2) which seeks to demonstrate the performance of the occulter mask for which the limiting factor is the occulter mask itself rather than such experimental limitations [12, 13].

The principal idea presented in this paper is a two-dimensional scalar diffraction propagation method that allows modeling of many types of experimental errors for external occulters. Simulation results are compared with laboratory results for the validation of the model to a 10⁻¹⁰ contrast level. Previous work deemed the direct two-dimensional evaluation of the Fresnel diffraction integral for occultor optical propagations to be impractically slow [14, 17]. Our approach combines the Matrix Fourier Transform (MFT) propagation method [15] with multi-frequency and fine structures to make the direct evaluation numerically tractable. Other optical propagation codes [7, 16–19] integrate along the occulter boundary and use analytical estimates of the impact of other types of errors (e.g., phase errors which are not expected to manifest at space-scale but can be a limiting factor for a laboratory experiment). By comparison, our method allows modeling all experimental errors simultaneously and evaluating their combined effect. The main contributions of this paper are:

- introducing a nomenclature in which we distinguish between suppression (at the telescope pupil) and contrast (at the telescope focal plane) and both are measured (Section 2).
- the development of a two-dimensional optical propagation model for external occulters (Section 3). We provide a validation numerical example demonstrating that the model can reproduce results of the commonly-used one-dimensional optical propagation codes (at the 10⁻¹³ contrast level that the occulter mask is capable of achieving by design).
- the application of this optical propagation model on the Princeton laboratory testbed (Section 4). We perform a sensitivity analysis to identify the effect on contrast performance of individual experimental errors including phase aberrations on the optical surfaces and volumetric scattering, optical axis misalignments, occulter mask tilt, and manufacturing accuracy of the occulter edges. Application of error parameters to levels expected in the laboratory environment is a good match to experimental results. We demonstrate in simulation that the factor limiting contrast performance (at the 10⁻¹⁰ level obtained in the laboratory experiment) is manufacturing accuracy of the occulter mask edges.
- validation of the hypothesis drawn from the sensitivity analysis that performance is limited by edge feature accuracy (Section 5). We perform an experimental demonstration involving an occulter mask manufactured with increased feature accuracy which results in measured suppression and contrast improvements. The measured contrast approaches the 10⁻¹² level – better than one order of magnitude improvement in contrast performance.

We conclude with a discussion of steps being undertaken for an experimental demonstration operating at conditions better approximating a space-mission (Section 6).

2. EXPERIMENTAL TESTBED AND RESULTS

A. Scaled Occulter Testbed

The apodization profile \( A(r) \) used in this study is shown in Figure 1a and its petalized realization that comprises the occulter mask is shown in Figure 1b with 0 being fully opaque and 1 fully transmissive. The inner region corresponds to the actual occulter which provides the optimized shadow, whereas the outer region corresponds to an apodized outer ring to mitigate deleterious diffraction effects from the mounting the occulter mask and the finite extent of the testbed. The optimized occulter apodization is designed using linear programming methods [7] with the optical propagation applying Babinet’s principle [26]. The outer ring apodization is the complement of a similarly-designed oversized occulter. The mask design process and equivalence to the occulter-only design is discussed in more detail in an earlier paper [20].

Because the occulter mask has finite radial extent due to the outer ring the optical propagation does not require usage of Babinet’s principle. The electric field \( E_{ap} \) past an occulter mask with circularly symmetric transmittance profile \( A(r) \) at a distance \( z \) downstream with wavelength \( \lambda \), can be written as the Fresnel integral [22]:

\[
E_{ap}(\rho) = \frac{2\pi}{i\lambda z} e^{\frac{i\pi\rho^2}{\lambda z}} \int_0^R e^{\frac{i\pi r^2}{\lambda z}} J_0 \left( \frac{2\pi \rho r}{\lambda z} \right) A(r)dr
\]

where \( R \) is the maximal radial extent of the mask, \( J_0 \) is the zeroth-order Bessel function of the first kind, \( \rho \) is the radial distance across the shadow, and \( r \) is the radial distance across the occulter mask. Optimization methods are used to obtain a transmission profile \( A(r) \) that produces a shadow with small amplitude magnitude at the pupil plane [7]. Petals (representing N-fold circular symmetry) are a binary approximation of the radial transmission pattern, and their effect is commonly modeled maintaining the one-dimensional integral via a Jacobi-Anger series expansion [7, 17].

Experimental verification of occulters on the ground requires scaling from space dimensions to laboratory dimensions while maintaining the validity of the optical propagation. The most common approach for occultor experiments is to introduce a scale factor that keeps the Fresnel numbers \( \frac{\rho^2}{\lambda z} \) constant, allowing the electric field downstream from the occulter to be
The artificial star is incident on an occulter mask with radius $R$ both situated on an optical table on the left-end of the enclosure. The shadow is cast a distance $z$ downstream from the occulter mask on the moveable camera-telescope in the right-end.

![Diagram of the occulter testbed](image)

**Fig. 2.** Schematic of the occulter testbed. The artificial star is incident on an occulter mask with radius $R$ both situated on an optical table on the left-end of the enclosure. The shadow is cast a distance $z$ downstream from the occulter mask on the moveable camera-telescope in the right-end.

![Image Plane, Log Scale](image)

**Table 1.** Summary of experimental parameters and their equivalent space scaling.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Laboratory</th>
<th>Space Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pinhole-to-occulter</td>
<td>1.5 m</td>
<td>Infinity</td>
</tr>
<tr>
<td>Propagation distance</td>
<td>9.1 m</td>
<td>97,000 km</td>
</tr>
<tr>
<td>Radius of dark zone</td>
<td>13 mm</td>
<td>12 m</td>
</tr>
<tr>
<td>Occulter diameter</td>
<td>44 mm</td>
<td>380 m</td>
</tr>
<tr>
<td>Outer ring diameter</td>
<td>88 mm</td>
<td>N/A</td>
</tr>
<tr>
<td>Inner working angle</td>
<td>8.4 arcmin</td>
<td>400 mas</td>
</tr>
<tr>
<td>Outer working angle</td>
<td>17 arcmin</td>
<td>800 mas</td>
</tr>
<tr>
<td>Telescope diameter</td>
<td>14 mm</td>
<td>17 m</td>
</tr>
<tr>
<td>Suppression band</td>
<td>400-1100 nm</td>
<td>400-1100 nm</td>
</tr>
</tbody>
</table>

The artificial star consists of a single-mode 2 mW HeNe laser operating monochromatically at 632 nm. To create the diverging beam, the beam is passed through two lenses acting as a beam expander, and focused onto a 15 µm pinhole through an off-axis parabolic mirror to spatially filter the surface errors on the optics (such aberrations are a limiting factor for collimated occulter experiments; see Section 4 for a simulated demonstration that this is not a factor in our experiment).

We note here this occulter design was generated to fit inside the propagation distance available in the testbed. Therefore, compared to a typical occulter design this corresponds to an oversized occulter (i.e., operates at a larger Fresnel number $\approx 600$, rather than the typical Fresnel number of 10-20). In practice this means that when imaged with the camera-telescope, the occulter appears overresolved compared to what we would expect for a space occulter. This is a useful property in terms of validating the occulter propagation model as it permits a better comparison and identification of stray light.

**B. Experimental Results**

We propose definitions for two key performance metrics for an occulter-based starlight suppression system. **Suppression** is measured directly at the pupil plane of the telescope. This measurement is taken by first centering the camera in the shadow of the occulter mask without the lens attached and taking long exposures; the mask is then removed and short exposures are...
taken with suppression obtained as the mean flux ratio between these across the aperture. Contrast is measured at the image plane with a camera focused on the artificial star pinhole, and is defined as the ratio between the flux at each pixel in the image formed when the mask is in place and the flux of the peak pixel of the point spread function without a mask. In the occulter literature, these terms are sometimes used interchangeably but it is useful to distinguish between them and measure both. This allows for verification of the consistency of the results; contrast measurements are spatially-dependent and can indicate limitations of the suppression performance of the occultor mask, as the source of any stray light can be directly observed.

In Figure 3, we present the reduced suppression and contrast experimental data sets obtained from the testbed which will be used in Section 4 to compare against simulated results from the diffraction model developed in Section 3. In Figure 3a, the contrast-calibrated image plane is shown with the red circles denoting the inner and outer working angles – this is the same data set previously reported in [20]. This data-set is collected to ensure no saturation occurs in individual frames. Calibrated ND filters are used to recover the high-contrast with respect to the artificial star. In the annular region, the best azimuthal median contrast recorded is $3.0 \pm 1.2 \times 10^{-10}$, with the worst contrast of $6.3 \times 10^{-9}$ occurring at the inner 50% throughput angle. We augment the previous contrast data set [20] with the corresponding suppression measurements. Figure 3a, the suppression-calibrated pupil plane mosaic is collected by stitching images collected with the exposed camera CCD at 5 mm intervals. The mean-recorded suppression is $1.4 \times 10^{-5}$.

By comparison, the designed (ideal) performance of the occultor mask is better than $10^{-13}$ contrast in the annular region with the brightest feature at the 50% throughput angle expected to be $1.3 \times 10^{-10}$. The corresponding theoretical mean suppression across the aperture is $1.2 \times 10^{-7}$.

3. TWO-DIMENSIONAL SCALAR DIFFRACITION MODEL

Here we develop a two-dimensional optical propagation model for obtaining both the suppression at the telescope pupil plane, and the contrast at the telescope focal plane downstream of an occultor mask. This method is direct and intuitive, and by comparison with existing one-dimensional propagation codes allows modeling any type of experimental error directly and assessing combined effects. We provide a numerical example using a high-resolution model that closely approaches the ideal performance of the laboratory occultor mask demonstrating that the model is accurate to very high contrast levels ($10^{-13}$).

A. Fresnel propagations between optical planes

We refer to Figure 4 to describe the propagations between the different planes involved in the occultor testbed. P0 is the plane of the pinhole source ($2M \times 2N$ in physical size), P1 is the plane of the occultor mask ($2U \times 2V$ in physical size), P2 is the pupil plane where the telescope aperture lies and where we measure the suppression of the shadow ($2X \times 2Y$ in physical size), and lastly P3 is the focal plane of the telescope and the plane in which we measure the contrast of the point spread function. Thus, to obtain the point spread function of the occultor testbed three separate Fresnel propagations between these four planes are needed. The numerical method used to perform the Fresnel propagations is MFT-based [15], which reduces the memory requirements compared to a traditional FFT-based propagation.

The first optical propagation occurs between the artificial star source and the occultor mask. The artificial star source has a pinhole with radius $r_{pin}$ across the plane P0 spanned by Cartesian coordinates $(m, n)$ and is defined by the following circular function:

$$A_{pin}(m, n) = \begin{cases} 1, & \sqrt{m^2 + n^2} \leq r_{pin} \\ 0, & \text{else.} \end{cases}$$

Then the input electric field at P1 spanned by Cartesian coordinates $(u, v)$ can be computed from a two-dimensional Fresnel integral over the pinhole at P0 as follows:

$$E_{in}(u, v) = \frac{2\pi n h}{\lambda} e^{i\pi/4} (u^2 + v^2) \int_{-N}^{N} \int_{-M}^{M} A_{pin}(m, n) \times \ldots$$

$$\int_{-U}^{U} \int_{-V}^{V} E_{in}(p, q) e^{i2\pi (mu + nv)/\lambda} \, dp \, dq$$

where $h$ is the propagation distance between P0 and P1, $-M < m < M$, $-N < n < N$, $-U < u < U$, and $-V < v < V$. This expression for $E_{in}$ assumes a uniform beam across the pinhole, but other profiles such as Gaussian can be chosen.

Next, the Fresnel propagation past the occultor mask represents the main diffractive propagation computing the shadow at P2 spanned by Cartesian coordinates $(x, y)$. We use a 2D mask with transmission profile $A(u, v)$ that represents the diffraction effect of the occultor mask. The resulting 2D Fresnel propagation integral between the occultor plane at P1 and the telescope’s pupil at P2 is as follows:

$$E_{pup}(x, y) = \frac{2\pi n h}{\lambda} e^{i\pi/4} (x^2 + y^2) \int_{-Y}^{Y} \int_{-X}^{X} E_{in}(u, v) \times \ldots$$

$$\int_{-U}^{U} \int_{-V}^{V} A(u, v) e^{i2\pi (ux + vy)/\lambda} \, du \, dv$$

where $z$ is the propagation distance between P1 and P2, $E_{in}$ is the output of the propagation in Equation 3, $-X < x < X$, and $-Y < y < Y$.

Finally, to propagate from the pupil plane at P2 to the image plane at P3 a quadratic phase function corresponding to a lens is introduced. Then, we perform a Fresnel propagation corresponding to a distance $s$ derived from the thin lens equation that takes into account the focal length $f$ of the lens and the distance $d$ to the pinhole that the focus of the lens is set to:

$$E_{img}(\xi, \eta) = \frac{1}{i\lambda s} \int_{-Y}^{Y} \int_{-X}^{X} E_{pup}(x, y) \times \ldots$$

$$\int_{-X}^{X} \int_{-Y}^{Y} A_{pup}(x, y) e^{i\pi/4} (x^2 + y^2) e^{i2\pi s (x_0^2 + y_0^2)} \, dx \, dy$$

where $A_{pup}$ is a circular function (similar to Equation 2) of radius matching the telescope’s aperture, $E_{pup}$ is the output of the propagation in Equation 4, and $(\xi, \eta)$ are Cartesian coordinates spanning the image plane P3.

To normalize the intensity at the pupil plane in terms of suppression or the intensity at the image plane in terms of contrast, we set $A(u, v) = 1$ and obtain $E_{pup}(x, y)$ as the electric field output of Equation 4 with no occultor mask in place and $E_{img}(\xi, \eta)$ as the corresponding electric field at the image plane from Equation 5. Then the suppression and contrast metrics are defined as follows:

$$\text{Supp.}(x, y) = \frac{|E_{pup}(x, y)|^2}{\max |E_{pup}(x, y)|^2}$$

$$\text{Cont.}(\xi, \eta) = \frac{|E_{img}(\xi, \eta)|^2}{\max |E_{img}(\xi, \eta)|^2}$$
Fig. 4. Optical planes for the occulter testbed. The diverging beam originates from the pinhole at P0 and is incident on the occulter mask at P1. The shadow is directly measured at P2 in terms of suppression, and the point-spread function of the system is measured at P3 in terms of contrast.

B. Occulter mask model generation

The mask model across the occulter plane P1 incorporates the mask manufacturing accuracy through the choice of rectangular grid discretization. Figure 5 illustrates the process of generating a two-dimensional occulter mask from the one-dimensional apodization profile.

The mask model is represented by the apodization profile \( A(u, v) \) in Equation 4. \( A(u, v) \) is set to white, or 1, in open areas in which light is allowed to pass and to black, or 0, in areas where the binary mask completely blocks light. To generate a binary mask we follow the procedure: 1) generate an \( n \times n \) grid; 2) determine polar coordinates of the midpoint of each grid point; 3) compute the radial and angular coordinates of a petal edge; 4) for each midpoint, compute the angular coordinate in relation to the nearest petal center; 5) determine whether each midpoint is a point on the mask (black) or an opening (white) by comparison to the nearest petal edge angular coordinate.

In Figure 5a, this process is shown graphically. A number of test points \( p1 - p7 \) are chosen that lie on the mask. These test points all have angular coordinates with respect to the petal center that are above the petal edge and are therefore set to black; the shaded area is above the petal edge and represents points on the mask that are fully opaque. Test points \( p8 - p12 \) have angular coordinates smaller than the petal edge and are therefore set to white and represent the petal openings; the entire white area is below the petal edge and represents points not on the mask that are fully transparent.

To perform a Fresnel propagation for a high resolution occulter mask, we utilize an anti-aliasing technique to reduce the number of grid points necessary to represent the occulter mask. We allow \( A(u, v) \) to take on gray values over the range \([0, 1]\) along the edges of the occulter mask in order to approximate a higher-resolution grid than can be numerically tractable to fully simulate as a binary mask. This is achieved by computing a pattern of white and black squares over an area and determining the black fraction of the total area to obtain a gray approximation. An example of this anti-aliasing procedure is shown in Figure 5b.

Thus, a mask of radius \( R \) with \( n \times n \) samples which uses \( g \times g \) anti-aliasing approximates a mask with feature size of \( \delta R = 2R/n/g \), where \( R \) is the radius of the mask to the outer edge. Such a mask with feature size \( \delta R = 0.55 \mu m \) is shown in Figure 5c.

C. Numerical example

To demonstrate the numerical accuracy of this diffraction model, we choose a detailed mask as shown in Figure 5c with \( n = 16,000, g = 10 \). At laboratory dimensions, this discretization approximates a feature size of \( \delta R = 0.55 \mu m \). We compare the propagation to the image plane for both the binary petalized mask, which represents a mask with perfect feature accuracy, with the performance of this high-resolution mask.

To compute the theoretical performance of a petalized occulter mask, a 1D propagation Fresnel diffraction integral has typically been used with a Jacobi-Anger expansion resulting in a series summation providing the ideal contribution from the petals as introduced in [7]. This represents the ideal performance of this testbed as designed. The computed contrast in the image plane is shown in Figure 6a, and the corresponding computed mean suppression across the telescope pupil is \( 10^{-0.91} \). The contrast in the annular working region reaches the \( 10^{-13} \) level.

Using the high-resolution 2D mask model from Figure 5c, the
Comparison of ideal model with finite feature-size limited model (a) Theoretical contrast at the image plane using the 1D binary propagation. Corresponding suppression is $2.0 \times 10^{-7}$. The inner and outer red circles denote the annular dark region. (b) Theoretical contrast at the image plane using a 2D propagation with a high-resolution mask featuring 0.55 µm feature size. Corresponding suppression is $2.5 \times 10^{-7}$.

Computed contrast result is shown in Figure 6b and attains a corresponding $10^{-6.60}$ mean suppression measurement, and the contrast in the annular working zone also reaches the $10^{-13}$ level. This comparison demonstrates that a mask with sufficiently high resolution can approach the ideal performance of the occulter mask as realized with 16-petals and validates the accuracy of the 2D propagation model to a deep contrast level.

4. SENSITIVITY ANALYSIS

The propagation model presented in the previous section does not contain any non-idealization errors except for discretization from the occulter mask grid. We can use this model to introduce many types of experimental errors individually, and combine these at expected laboratory levels.

In the pupil plane, the suppression of the experimental mask levels off across the dark hole (see in Figure 3b). However, we can use the image plane results in Figure 3a, where we can identify the provenance of the additional light leakages. The bright edges are most likely caused by diffraction from departures from the ideal occulter shape (for example localized manufacturing defects in the form of over or under-etching). Misalignment of the input beam manifests as a bright ring around the inner occulter mask. Volumetric scattering from aerosol contamination can produce phase shifts across the aperture while surface aberrations from the artificial star optics are expected to be filtered by the pinhole (a limiting effect for collimated experiments).

A. Manufacturing process feature size

When converting the mask’s apodization profile for CAD specifications for etching of the silicon wafer, the output consists of 16 different sets of points with each defining a polygon petal opening of the occulter mask. The spacing of points along the petal edges represents the accuracy with which the polygon is defined – a small number of points results in longer straight edges which can introduce additional diffraction leakage along the occulter edges.

For the 2D diffraction model previously described in Section 3, the manufacturing process δR feature size can be represented by choice of the discretization grid of the occulter mask via the number of sampling points n and the number of anti-aliasing points g. A number of different occulter mask models are generated to represent masks with different uniform feature sizes.

Using these mask models across the occulter plane P1, we perform optical propagations and compare these to the high-resolution mask performance. Figure 7 shows the contrast-calibrated image plane results as the manufacturing feature size is worsened. Suppression performance across the dark hole is reduced from $2.5 \times 10^{-7}$ for the high-resolution mask to $4.1 \times 10^{-5}$ in the worst case shown. The corresponding contrast across the dark annular regions in the image plane is decreased from $7.8 \times 10^{-13}$ to $3.9 \times 10^{-11}$.

Qualitatively we can see in the image plane that the edge diffraction is significantly increased resulting in bright edges across the occulter mask similar to those seen experimentally in Figure 3a. A clear trend in Figure 7 is that larger feature size result in significantly more diffraction across the mask edges than predicted from the perfect petal realization expected from the ideal theoretical performance.

B. Edge perturbations

The resulting two-dimensional mask $A(u, v)$ after choice of discretization parameter is symmetrical (except for discretization errors arising from representation of a 16-fold circularly symmetric pattern onto a two-dimensional grid). To remove symmetry and simulate manufacturing variations between different petals of the occulter mask, we consider the effect of a set of circular defects added along the petal edge. These defects can either be under-etching (reduction in transmission) or over-etching (increase in transmission).
To obtain an estimate of the amount of perturbations to realistically introduce in the model, we performed spot microscope imaging of the occulter mask. The procedure to obtain the microscope images and fit them to the designed occulter edges are described in more detail in [13, 24]. The deviation between the measured edge and the theoretical contour is obtained for each measured point along the extracted edge and we obtained an estimated RMS value of 3.1 μm.

We model edge perturbations as a set of circular defects that are added (for under-etching) or subtracted (for over-etching) from the occulter mask’s edge. We first identify 16 different sets of continuous gray contour grid points indexed by k across each petal \( C^k = \{ \epsilon^k \mid 0 < \epsilon^k < 1, \epsilon \in \mathbb{R} \} \) with each set containing \( N_k \) elements representing the number of grid points along the petal contour.

Using the procedure described in [25], 16 different stochastic sequences given by \( E^k = \{ \epsilon^k \mid \epsilon^k \in \mathbb{R} \} \) are each injected in a different petal contour. Each entry in the set represent the radius of one circular defect. The stochastic sequence is generated as a set of random Gaussian variables with unity variance representing discrete spatial frequency components up to the Nyquist limit, then multiplied with a 1/f\(^n\) envelope. The DC component is set to 0, and to represent white noise we set \( a = 0 \). The sequence is then converted via an FFT, using appropriate zero-padding, to the spatial domain across the entire petal and normalized to a RMS-value matching the measured deviation.

Next, we model the addition of the set of circular defects onto the original petal contour. The set of pixels along the petal representing the occulter edge is \( \bar{C}^k = \{ \epsilon^k \mid 0 \leq \epsilon^k \leq 1, \epsilon^k \in \mathbb{R} \} \) for each petal \( k \). We then add the effect of the circular sequence to the original petal contours:

\[
e^k_i = c^k_i + \frac{\pi(e^k_i)^2}{(2\alpha\pi R)^2}, \quad \forall i \in \{1 \ldots N_k\} \tag{8}
\]

For large circular defects, we maintain the condition that \( 0 \leq \epsilon^k_i \leq 1 \) to avoid unphysical effects. The large defect is instead bled into neighboring pixels using the same approach by estimating the fractional area that the defect spans across these pixels.

In Figure 8 we show the contrast-calibrated image plane results for the high-resolution occulter mask with increasing amounts of RMS edge perturbations injected as described. Qualitatively, we see that with increasing edge perturbations there is increased diffraction along the occulter edges. Diffraction from the edges is also very uniform, a consequence of the white-noise structure of the injected perturbations.

### C. Input beam modeling

The non-uniform amplitude profile of a Gaussian beam across the pinhole coupled with displacement of the optical axis of the pinhole from the occulter plane to simulate optical misalignment is a departure from the uniform amplitude profile assumed as illuminating the occulter for the original space design. Additionally, we adopted a diverging beam to eliminate optical aberrations from optical surfaces. Therefore, it is important to investigate the sensitivity at laboratory conditions for all such input beam errors.

To account for the Gaussian beam and optical aberrations across the pinhole, we can modify Equation 3 which was written for a uniform beam across the pinhole by introducing two additional multiplicative terms inside the integral

\[
E_{\text{in}}(u, v) = \frac{e^{2\pi i k/\lambda}}{iA h} e^{\frac{\pi}{\lambda} (u^2+v^2)} \int_{-N}^{N} \int_{-M}^{M} E_{\text{beam}}(m, n) e^{i\phi_{\text{pin}}(m, n)} \times \ldots A_{\text{pin}}(m, n) e^{\frac{\pi}{\lambda} (m^2+n^2)} e^{\frac{-\alpha}{2\alpha} \sqrt{(mu+nv)} dmdn} \tag{9}
\]

where \( \phi_{\text{pin}}(m, n) \) are the phase aberrations introduced by the folding mirror and focusing optics, simulated as pure phase aberrations collocated at the pinhole plane. A circularly symmetric Gaussian beam can be written at the pinhole plane as:

\[
E_{\text{beam}}(m, n) = e^{-\frac{\pi}{\alpha} \sqrt{m^2+n^2}} \tag{10}
\]

Here we use the complex beam parameter \( q \) and ABCD ray transfer matrices to relate \( q \) at the laser focus with propagation through free-space and subsequent focusing. The resulting Gaussian beam profile across the 15 μm diameter pinhole can have added phase aberrations from the upstream optics of RMS \( \lambda/4 \).

Lastly, we displace the center of the of the input pinhole with respect to the occulter mask to simulate misalignment of directions transverse to the optical axis (we note here that occluders are very robust to misalignments along the optical axis).

In Figure 9, we show the contrast-calibrated image plane sensitivity analysis results. We first introduce a pure beam misalignment in the diagonal direction as shown in Figure 9a. This results in light leakage around the central portion of the mask as seen experimentally, with performance relatively robust until there is a clear misalignment and a large amount of light leaks around the central occulter mask.

Next, we introduce pure phase aberrations across the pinhole plane P0, with the results shown in Figure 9b for increasing phase aberrations. These aberrations are produced similarly to the procedure described for generating the one-dimensional random edge perturbations with two main differences: we generate a two-dimensional aberrated matrix instead of a one-dimensional sequence, and instead of a flat frequency envelope used for white noise we apply coloured noise with a frequency envelope of \( 1/f^\alpha \) where \( \alpha = \frac{3}{2} \). Because of the spatial filtering effect of the pinhole, high-frequency phase aberrations from the
The occulter mask is tilted at a $5^\circ$ angle to ensure a ghost reflection does not back-propagate into the occulter plane. Previous tolerancing studies have shown that occulters are tolerant of out-of-plane tilts [19, 23]. These studies have applied a tilt on the shadow which is a good first-order approximation, but using the two-dimensional mask model we can model the loss of symmetry directly due to the mask tilt (and combine this with other errors) by rotation and in-plane projection of the occulter mask.

In Figure 10, we show contrast-calibrated image plane results for increasing mask tilts. We can see that the performance of the mask is very robust to the loss of amplitude symmetry due to the mask tilt. Suppression performance only decreases to $3.3 \times 10^{-7}$ for a $20^\circ$ tilt (compared to an unaberrated $2.5 \times 10^{-7}$ suppression). There is more variation for contrast levels across the dark annular regions than is directly apparent from the suppression measure, primarily arising due to the elliptical change of the image shape.

![Fig. 10. Contrast as a function of mask tilt](image)

E. Camera model

We have developed a camera model to apply realistic measurement errors in simulations. The camera model consists of inclusion of: (1) optical aberrations on the imaging lens and (2) shot noise injected into the simulated image.

We have added optical aberrations on the simulated camera lens. These aberrations are generated similarly to the phase aberrations described previously in Section C with a given standard deviation $c_{\text{cam},\text{RMS}}$ and a decreasing $f^{-3/2}$ frequency ramp representative of optical aberrations. These are added as an additional phase term across the camera pupil at plane P2 in Equation 5. In Figure 11, we show the effect on the image plane as the standard deviation of the phase aberrations added across the lens is increased. The aberrations must be very large to be dominant. For expected aberrations of $\lambda/4$ or smaller the effect of introducing phase aberrations on the camera telescope is not significant; this demonstrates what has been widely considered to be a strength of external occulters, that is insensitivity to the optical quality on the telescope [4].

Inclusion of camera noise is particularly relevant to the contrast-calibrated image-plane simulations which feature high-dynamic range between the glowing occulter edges and the dark regions between the petals. Gaussian shot noise is injected into the simulated image to match the experiment using contrast-calibrated reduced camera dark frames.

![Fig. 11. Contrast as a function of camera lens aberrations.](image)

F. Comparison to experimental results

We combine all the different modeled errors in one optical propagation to assess the diffractive model performance of the occulter mask in the laboratory environment. Realistic parameters for the laboratory environment are listed in Table 2. The feature and edge accuracy are chosen to match the spot microscope images. Additionally, it is known that the mask does not have fully uniform feature sizes – these increase at the inner and outer tips where the apodization changes most rapidly. We introduce radially-dependent perturbation of the edges to match these larger feature changes. The mask tilt is a measured laboratory parameter. The phase aberrations across the pinhole are due to the vendor-quoted surface quality of the upstream optics. The beam displacement is measured from the experimental image plane results. The wavefront aberrations across the occulter plane are unknown, however we choose a small value as we can infer from a qualitative comparison of the experimental results.

### Table 2. Summary of realistic error parameters for diffractive simulation of laboratory environment.

<table>
<thead>
<tr>
<th>Error Parameter</th>
<th>Estimated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feature size, $\delta R$</td>
<td>2.41 µm</td>
</tr>
<tr>
<td>Edge perturbations</td>
<td>3.1 µm RMS</td>
</tr>
<tr>
<td>Optics aberrations, $c_{\text{pin},\text{RMS}}$</td>
<td>$\lambda/4 \approx 160$ nm RMS</td>
</tr>
<tr>
<td>Wavefront aberrations, $c_{\text{occ},\text{RMS}}$</td>
<td>3 nm RMS</td>
</tr>
<tr>
<td>Camera aberrations, $c_{\text{cam},\text{RMS}}$</td>
<td>30 nm RMS</td>
</tr>
<tr>
<td>Diagonal beam misalignment</td>
<td>4.1 mm</td>
</tr>
<tr>
<td>Mask tilt</td>
<td>$5^\circ$</td>
</tr>
</tbody>
</table>

Optics are filtered out and only low-order aberrations such as tip-tilt remain. These can be clearly seen in the image plane results for large aberrations on the order of $\lambda/4$ or greater, but these have a small effect on the performance metrics. For example, the worst suppression performance is only $3.0 \times 10^{-7}$ compared to the unaberrated performance of $2.5 \times 10^{-7}$.

We can also use this model to demonstrate the deleterious effect of phase errors on the occulter mask, thus validating the choice of a diverging beam in this experiment. We introduce stochastic phase errors collocated at the occulter plane P1 with a frequency envelope of $f^{-3/2}$, typical of optical surface errors. The sensitivity to stochastic phase errors at the occulter mask plane with increasing RMS is shown in Figure 9c. In our experiment, volumetric scattering from propagation through air (rather than vacuum) can introduce aberrations between the pinhole and the occulter plane. These were expected to be insignificant because of the short distance between the pinhole and the occulter plane. These expected to be insignificant because of the short distance between the pinhole and the occulter plane. These were expected to be insignificant because of the short distance between the pinhole and the occulter plane. Therefore, we can infer from a qualitative comparison of the experimental results seen in the sensitivity simulations here.)
to the sensitivity analysis that the wavefront aberrations are not a suppression-dominating limiting factor. Finally, the camera aberrations are a second-order effect under $\lambda/4$ which is above the expected imaging lens quality.

In Figure 12, we compare a contrast-calibrated image plane derived from the simulated optical propagation including all the modeled errors in Figure 12a with the experimental results (previously shown in Figure 3a) side-by-side for ease of comparison.

For a more quantitative comparison, the azimuthal cross-sections of contrast-calibrated image plane results are shown in Figure 13 and we also include the theoretical curve for the ideal, unaberrated 16-fold circular realization corresponding to Figure 6a. As can be seen from the figure, the introduction of error parameters corresponding to the expected values in the testbed demonstrates good agreement to the measured contrast curve. The dominating factor in terms of performance is the combination of finite feature sizes in the manufacturing of the mask and deviations from the prescribed ideal shape.

Some of the errors introduced, including the edge perturbations, the optical train aberrations prior to the pinhole, and phase aberrations collocated at the occulter plane are stochastic errors.

![Figure 12](image12.png)

**Fig. 12.** (a) Experimental contrast at image plane. Corresponding measured suppression $1.5 \times 10^{-5}$. (b) Diffractive analysis with experimental errors. Corresponding modeled suppression $1.7 \times 10^{-5}$.

![Figure 13](image13.png)

**Fig. 13.** Azimuthal median cross-section comparison between the experimental results, the theoretical performance of the idealized occulter mask, and the performance with the diffractive modeled error sources included.

The occulter performance can vary between simulations. We therefore perform a Monte Carlo assessment using a larger number of simulations with stochastic errors generated randomly for each of the aberrations described above. Such an ensemble of simulations gives a better assessment of the variation of the performance of the occulter than a single simulation. In Figure 14 we show the suppression results from a set of 100 simulations at 633 nm that provides an estimate of the expected suppression distribution. The suppression results are placed into ten equally sized bins, with the simulation frequency indicated. The probability density function (PDF) of the expected suppression performance is reconstructed from the simulated suppression results through normal kernel density estimates [27, 28] (using the `ksdensity` function in MATLAB’s Statistics toolbox). We can obtain quantitative estimates of the simulated suppression performance by computing the cumulative distribution function (CDF). From the CDF, we find the expected model suppression to be $1.0 \times 10^{-5}$ at the 5th percentile and $1.7 \times 10^{-5}$ at the 95th percentiles.

Thus, for expected laboratory parameters, the measured suppression performance falls within the simulated distribution. The contrast curves are well matched at the peaks which dominate the measured suppression. We have identified the limiting factors being primarily driven by diffraction from the mask edges.

![Figure 14](image14.png)

**Fig. 14.** Suppression performance for a Monte Carlo analysis using 100 simulations at 633 nm and the corresponding probability density function. Expected suppression performance is $1.0 \times 10^{-5}$. The 5th percentile suppression is $7.8 \times 10^{-6}$ and the 95th percentile suppression is $1.7 \times 10^{-5}$. The measured suppression of $1.5 \times 10^{-5}$ falls within the 95% confidence interval.

5. EXPERIMENTAL VALIDATION

To validate the hypothesis of the sensitivity analysis in the previous section, we have manufactured an improved occulter mask using better edge feature accuracy and smaller deviations from the prescribed shape and tested this new mask in the testbed. This required changing the original DRIE process with electron beam lithography and further thinning of the sidewalls, necessary to better control the edge feature accuracy. This improved manufacturing process has been developed primarily for coronagraph masks for the WFIRST mission and is described in more detail in a forthcoming publication.
6. CONCLUSIONS

In this paper, we have proposed a direct two-dimensional diffraction model to evaluate occulter performance. We have shown that this model is able to replicate the ideal, unaberrated performance predicted by extant one-dimensional propagation codes at the $10^{-13}$ contrast level and the $10^{-7}$ suppression level. Additionally, we have compared the model against experimental data and were able to replicate measured contrast to a level better than $10^{-10}$ and suppression to a level approaching $10^{-5}$ using a combination of errors with parameters matching expected laboratory levels.

This analysis has also shown that the dominating error terms for the testbed are occulter edge effects. An experimental validation of the hypothesis drawn from the sensitivity analysis demonstrated that manufacturing improvements in the edge feature accuracy of the occulter mask improve both measured contrast and suppression performance. The measured contrast level is approaching the $10^{-12}$ level and represents a better than one order of magnitude improvement compared to the original occulter mask, while the suppression level improved by a factor of four.

For an experimental demonstration of occulter performance at a space geometry, we will need to operate at much smaller Fresnel numbers (in the range of 10-20). The current testbed is limited in terms of the propagation distance, but we have used our experimentally-verified diffraction model to design and predict the performance of a more realistic testbed operating at typical flight Fresnel numbers by expanding the testbed propagation distance [13, 30]. In the existing testbed the enclosure was oversized guaranteeing no deleterious diffraction effects. For an extended testbed this will not be possible due to the larger propagation distances involved. We have found the diffraction model presented here useful for estimating the effect of a type of experimental error previously unmodelled – diffraction effects due to the finite tunnel size which will be the subject of a future communication. This extended testbed is currently under construction at Princeton [31], and when operational it will enable experimental measurements of suppression approaching $10^{-9}$ while maintaining contrast better than $10^{-10}$ as required for exo-Earth imaging.

FUNDING INFORMATION

This work was partially performed under NASA contract NNX09AB97G and grant 1430187 from the California Institute of Technology’s Jet Propulsion Laboratory. DS acknowledges financial support from the NASA Earth & Space Science Fellowship and the Natural Sciences and Engineering Research Council of Canada postgraduate scholarship.

Fig. 15. (a) Experimental contrast at image plane for the occulter mask with improved feature accuracy. Corresponding measured suppression is $6.0 \times 10^{-6}$ (b) Simulated contrast at the image plane.

Fig. 16. Azimuthal median cross-section comparison between the experimental results corresponding to the original mask (solid blue) with the experimental results corresponding to the mask featuring improved feature accuracy (solid green). These are compared with the theoretical performance of the idealized occulter mask (black) and the performance with the diffractive modeled error sources for the improved feature accuracy mask (dashed green).
ACKNOWLEDGMENTS

We thank Bala Balasubramanian, Eric Cady, Pierre Echternach, Tyler Groff, AJ Riggs, Stuart Shaklan, and Victor White for many helpful discussions.

REFERENCES


