On Designing NASA's Terrestrial Planet Finder Space Telescope

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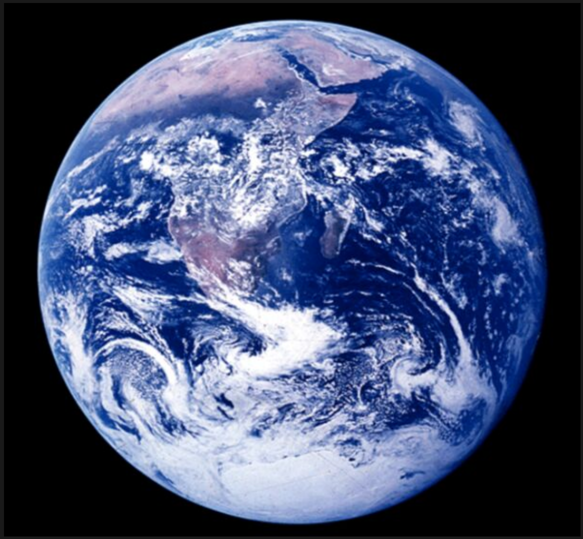
The Big Question: Are We Alone?

- Are there Earth-like planets?
- Are they common?
- Is there life on some of them?



Exosolar Planets—Where We Are Now

There are more than 100 Exosolar planets known today.

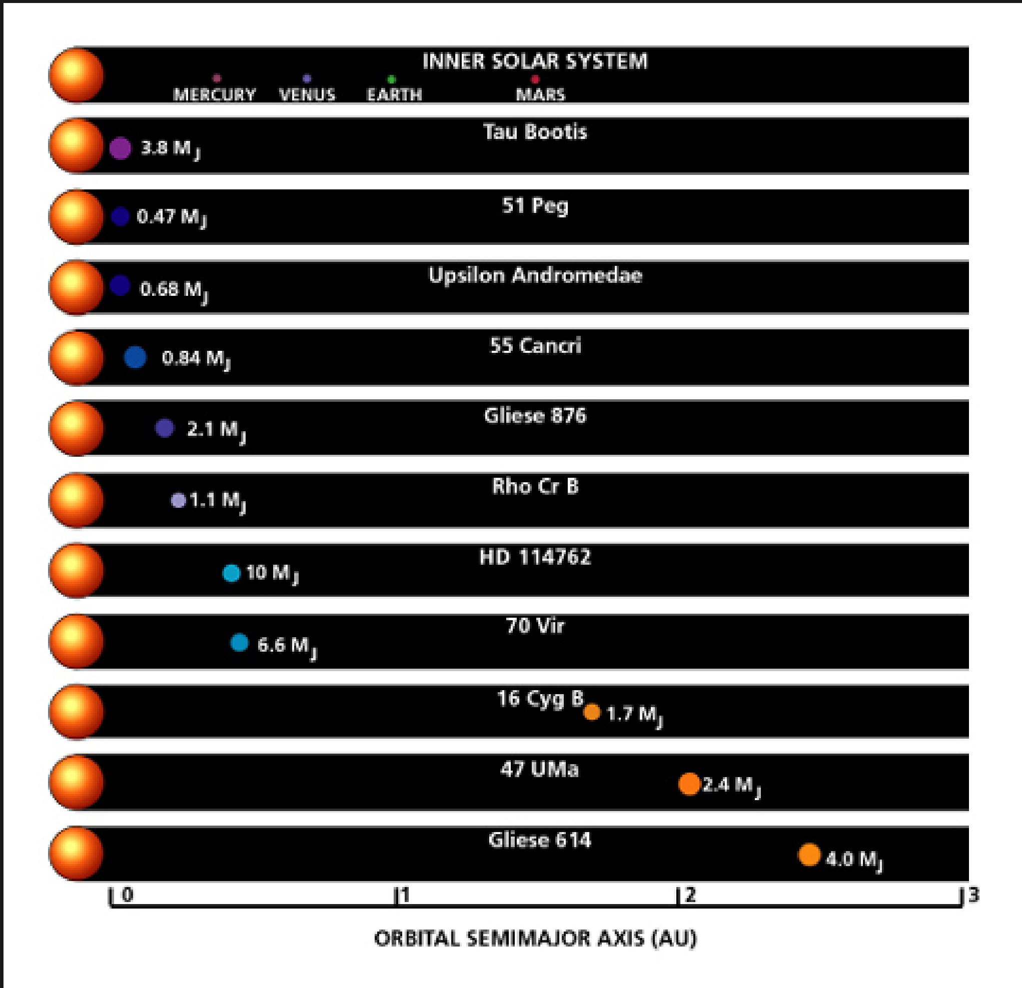
Most of them have been discovered by detecting a sinusoidal doppler shift in the parent star's spectrum due to gravitationally induced wobble.

This method works best for large Jupiter-sized planets with close-in orbits.

One of these planets, HD209458b, also transits its parent star once every 3.52 days. These transits have been detected photometrically as the star's light flux decreases by about 1.5% during a transit.

Recent transit spectroscopy of HD209458b shows it is a gas giant and that its atmosphere contains sodium (as expected).

Some of the ExoPlanets



Terrestrial Planet Finder Telescope

- DETECT: Search 150-500 nearby (5-15 pc distant) Sun-like stars for Earth-like planets.

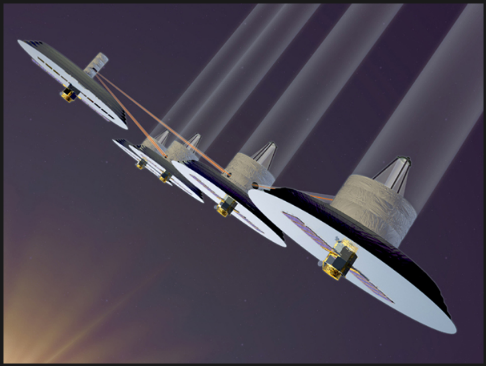
- CHARACTERIZE: Determine basic physical properties and measure "biomarkers", indicators of life or conditions suitable to support it.

Why Is It Hard?

- If the star is Sun-like and the planet is Earth-like, then the visible light from the star is 10^{10} times brighter than the reflected light from the planet. This is a difference of 25 magnitudes!

- If the star is 10 pc (33 ly) away and the planet is 1 AU from the star, the angular separation is 0.1 arcseconds!

Originally, it was thought that this would require a space-based multiple mirror nulling interferometer.

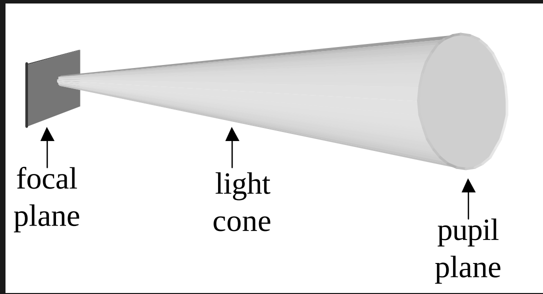


However, a more recent idea is to use a single large telescope with an elliptical mirror (4 m x 10 m) and a *shaped pupil* for diffraction control.

HD209458 is the bright (mag. 7.6) star in the center of this image.
The dimmest stars visible in this image are magnitude 16.
An Earth-like planet 1 AU from HD209458 would be magnitude 33, and would be located 0.2 pixels from the center of HD209458.

The Shaped Pupil Concept

Consider a telescope. Light enters the front of the telescope—the *pupil plane*.



The telescope focuses the light passing through the pupil plane from a given direction at a certain point on the *focal plane*, say $(0, 0)$.

However, the wave nature of light makes it impossible to concentrate all of the light at a point. Instead, a small disk, called the *Airy disk*, with diffraction rings around it appears.

These diffraction rings are bright relative to any planet that might be orbiting a nearby star and so would completely hide the planet. Our Sun, for example, would appear 10^{10} times brighter than Earth to a distant observer.

By placing a mask over the pupil, one can control the shape and strength of the diffraction rings. The problem is to find an optimal shape so as to put a very deep *null* very close to the Airy disk.

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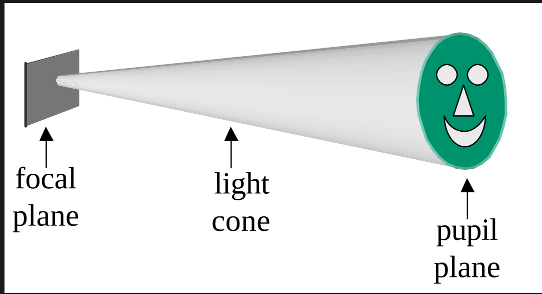

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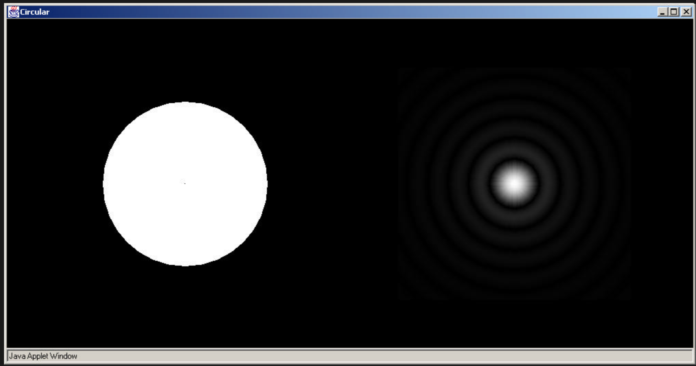
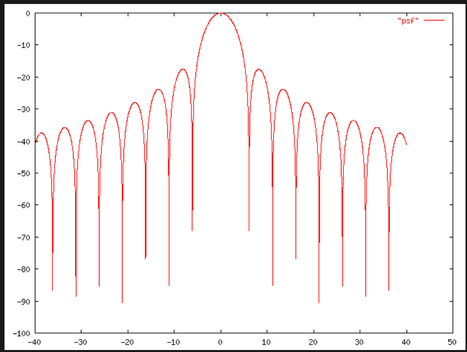
Airy Disk and Diffraction Rings

A conventional telescope has a circular openning as depicted by the left side of the figure. Visually, a star then looks like a small disk with rings around it, as depicted on the right.



The rings grow progressively dimmer as this log-plot shows:



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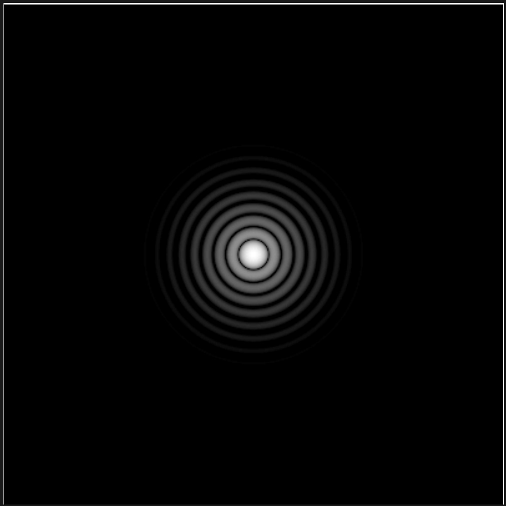
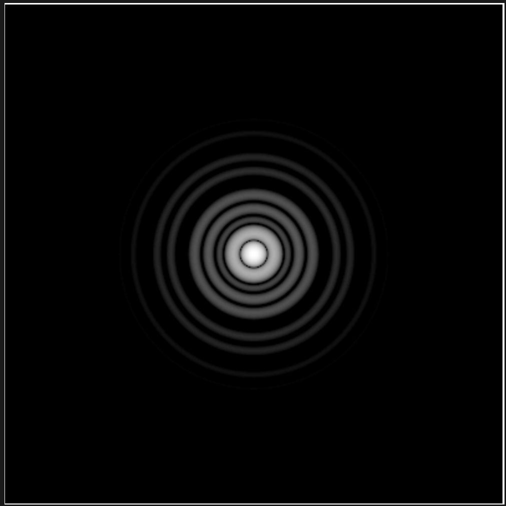
Central Obstructions are an Example of a Shaped Pupil

Logarithmically scaled plots of 2-D point spread functions for apertures with and without a 30.3% central obstruction. White is 1 and black is 10^{-4}.

Without (refractor):

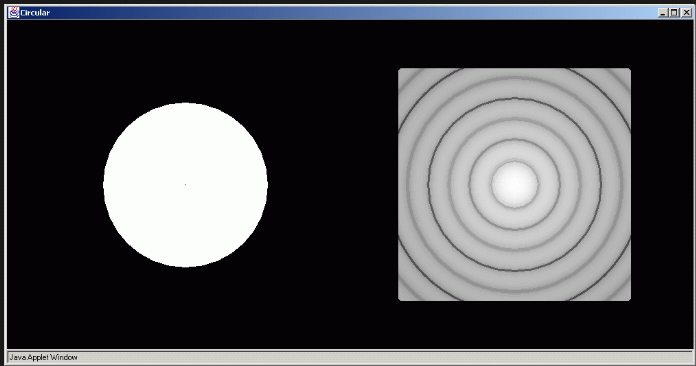


With (Questar):



Airy Disk and Diffraction Rings—Log Scaling

Here's the unobstructed Airy disk from the previous slide plotted using a logarithmic brightness scale with 10^{-11} set to black:



The problem is to find an aperture mask, i.e. a pupil plane mask, that yields a 10^{-10} dark zone somewhere near the first diffraction ring. *A hard problem!* Such a dark zone would appear almost black in this log-scaled image.

Mathematical Details

The light from a distant point source, i.e. a star or a planet, is a coherent plane wave.

A coherent plane wave passing the pupil plane produces an interference pattern at the focal plane given by the 2-D Fourier transform of the indicator function of the mask:

$$E(\xi, \zeta) = \iint e^{-ik(x\xi + y\zeta)/f} 1_{\mathcal{M}}(x, y) dy dx.$$

Here, k is the wave number $(2\pi/\lambda)$, f is the focal length of the instrument, and $\mathcal{M}$ denotes the set where the mask is open.

We assume that $\mathcal{M}$ is symmetric wrt to the axes so that E is real.

What is measured at the image plan is the square of the magnitude of the electric field:

$$P(\xi, \zeta) = |E(\xi, \zeta)|^2$$

Note that $E(0, 0)$ is the open area of the mask—i.e., the throughput.

The Optimization Problem

The problem is to maximize light throughput, i.e. the open area of the mask, subject to the constraint that the intensity of the light in a specified dark zone $\mathcal{S}$ is at most 10^{-10} as bright as at the center of the star's Airy disk:

$$\begin{array}{rll} \text{maximize:} & E(0,0) & \\ \text{subject to:} & -10^{-5}E(0,0) \leq E(\xi,\zeta) \leq 10^{-5}E(0,0), & x \in \mathcal{S} \\ & \mathcal{M} \subset \mathcal{A} & \end{array}$$

Here, $\mathcal{A}$ denotes the shape of the underlying mirror—typically, a circle or an ellipse.

Specifying $\mathcal{M}$

If the set $\mathcal{M}$ is given by a function M:

$$\mathcal{M} = \{(x, y) : -M(x) \le y \le M(x)\}$$

then the optimization problem is infinite dimensional and nonlinear.

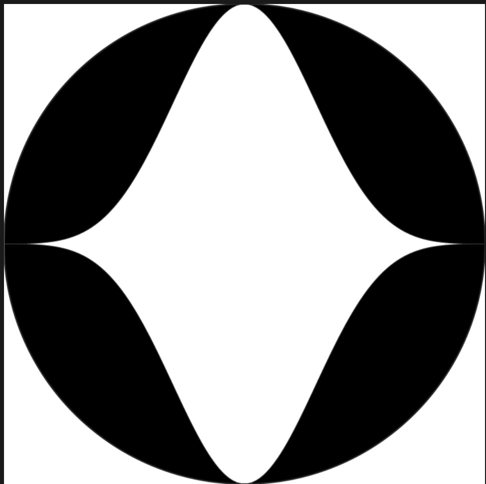
In this case, the electric field can be written as

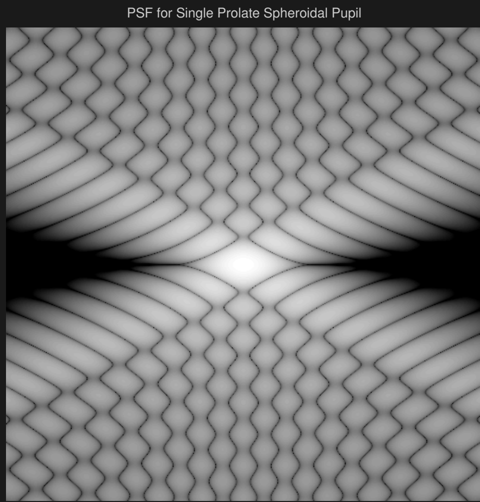
$$E(\xi, \zeta) = 2 \int_{-a}^{a} \cos(kx\xi/f) M(x) \mathrm{sinc}(k\zeta M(x)/f) dx.$$

Nonetheless, discretizing the x-axis and the set $\mathcal{S}$, one can reduce the problem to a finite dimensional problem that can be solved with modern NLP software.

A Linear Programming Problem

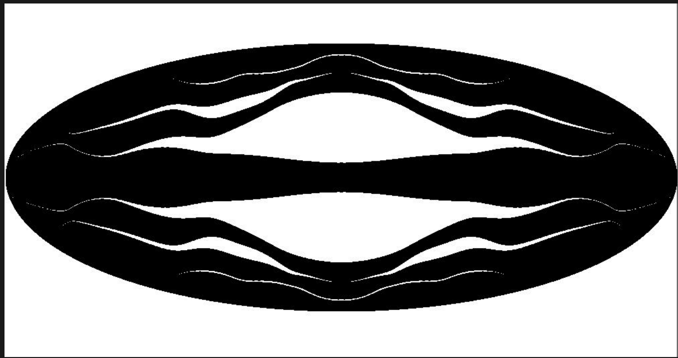
Taking the set $\mathcal{S}$ to be a subset of the ξ-axis, $\mathcal{S} = \{(\xi, 0) : |\xi| \geq 4\}$ the electric field becomes linear in $M(x)$ and so the problem becomes a linear programming problem.

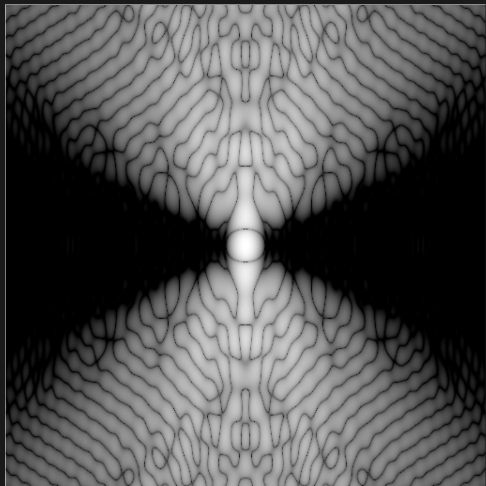




Unfortunately, the discovery zone is too cusp-like close in to the Airy disk to be of use.

Multiple Openning Masks





Azimuthally Symmetric Mask

If we choose $\mathcal{M}$ to be rotationally invariant, then the mask consists of several concentric rings

$$\begin{array}{ll} [r_0, r_1] & \text{first opening} \\ [r_2, r_3] & \text{second opening} \\ & \vdots \\ [r_{2m-2}, r_{2m-1}] & m\text{-th opening} \end{array}$$

In this case, the electric field is a function only of radius $\rho = \sqrt{\xi^2 + \zeta^2}$:

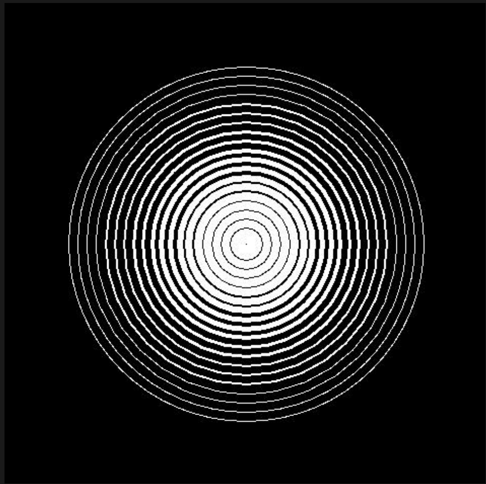
$$\begin{aligned} E(\rho) &= 2\pi \sum_{k=0}^{m-1} \int_{r_{2k}}^{r_{2k+1}} J_0(2\pi r \rho) r dr, \\ &= \frac{1}{\rho} \sum_{k=0}^{m-1} \left(r_{2k+1} J_1(2\pi r_{2k+1}\rho) - r_{2k} J_1(2\pi r_{2k}\rho) \right) \end{aligned}$$

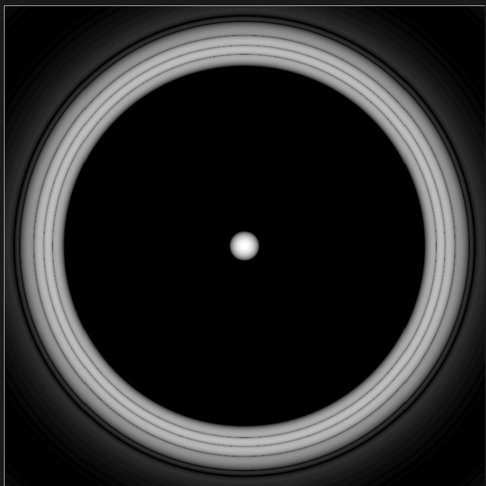
And the open area can be written as a sum of differences of squares:

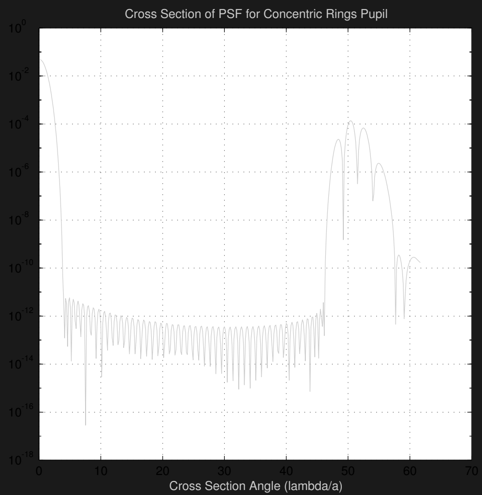
$$\sum_k \left(r_{2k+1}^2 - r_{2k}^2 \right).$$

Highly nonconvex, nonlinear optimization. Yet, it can be solved...

Azimuthally Symmetric Mask







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Apodization

Instead of cutting a mask, how about tinting glass? This is called apodization. Given an apodization function A, the electric field is

$$E(\xi, \zeta) = \iint e^{-ik(x\xi + y\zeta)/f} A(x, y)\,dy\,dx.$$

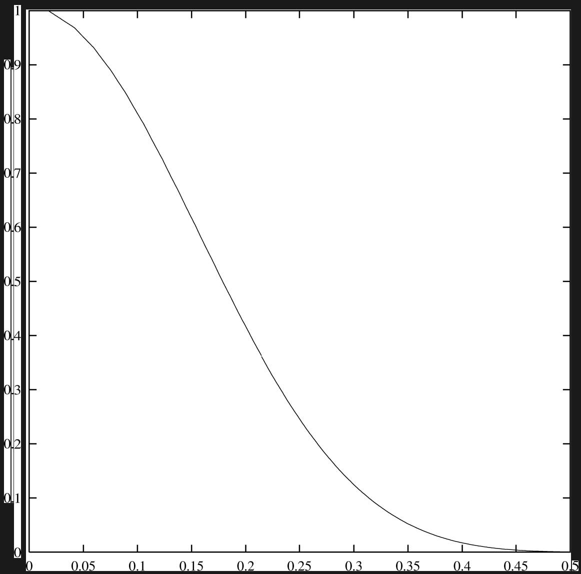
If the apodization is rotationally invariant, then so is the electric field:

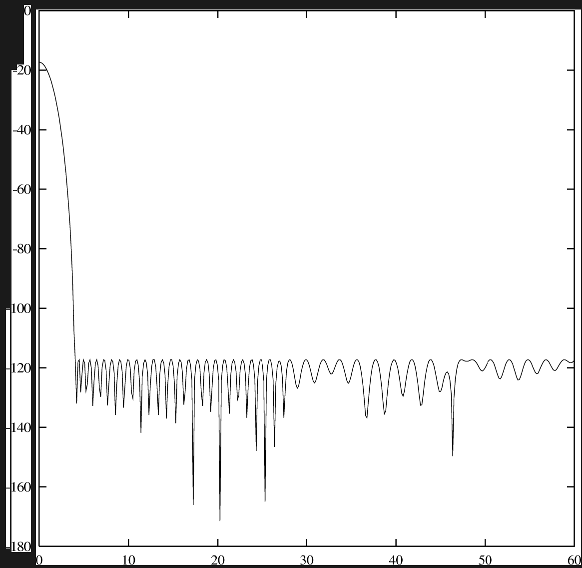
$$\begin{aligned} E(\rho) &= \int_0^{1/2} \int_0^{2\pi} e^{-2\pi i r\rho\cos(\theta-\phi)} A(r)\,r\,d\theta\,dr, \\ &= 2\pi \int_0^{1/2} J_0(2\pi r\rho) A(r)\,r\,dr, \end{aligned}$$

where J_0 denotes the 0-th order Bessel function of the first kind.

Note that the mapping from apodization function A to electric field E is linear.

Optimal Solution (with smoothness constraints)





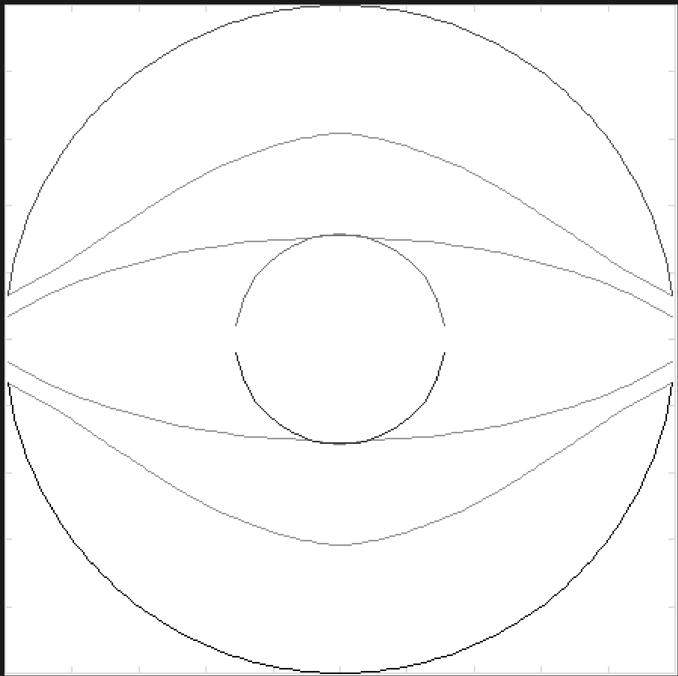
Two drawbacks:

1. It is currently not possible to manufacture such an apodization to the required precision.
2. For apodization, throughput is the integral of the *square* of the apodization function, which typically is about half the integral of the function itself.

Field Test

Dim Double Splitter Mask

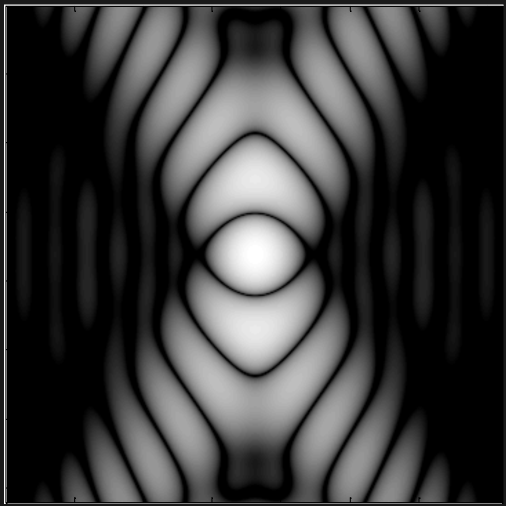
A mask was made for a 3.5" Questar. The mask was cut from paper with scissors (a crude tool at best) according to the template shown, backed with cardboard, and framed with 4" PVC endcap.

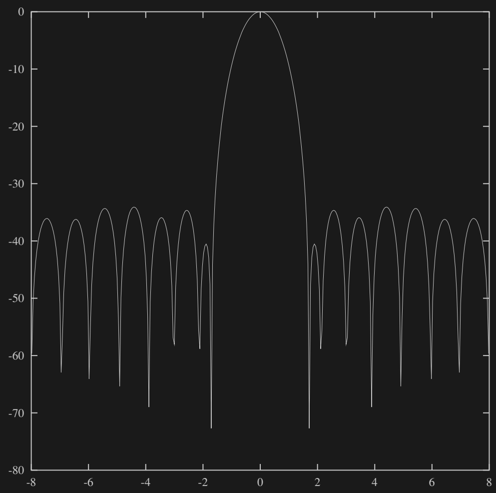


The outer circle represents the full aperture, the inner circle the central obstruction, and the remaining arcs the mask openning.

Computed PSF

Logarithmically scaled plot of the 2-D point spread function and a graph of its x-axis slice. White is 0 dB and black is -40 dB. Throughput is 18.2%.

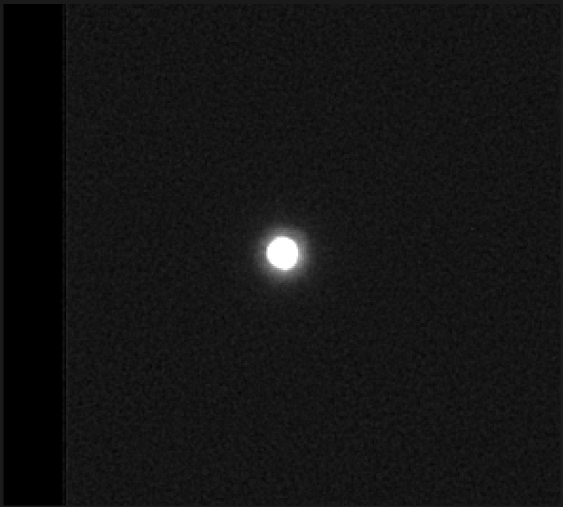




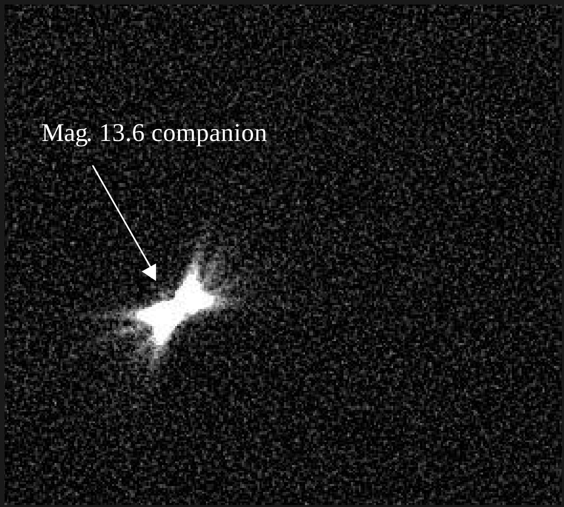
31 Leonis

31 Leonis is a dim double.
Primary/secondary visual magnitude: $4.37/13.6$
Luminance difference $= 9.2 = -36.8$ dB
Separation: $7.9" = 6.9\lambda/D$ (at 500nm). Position Angle: $44°$

Without mask:



With mask:

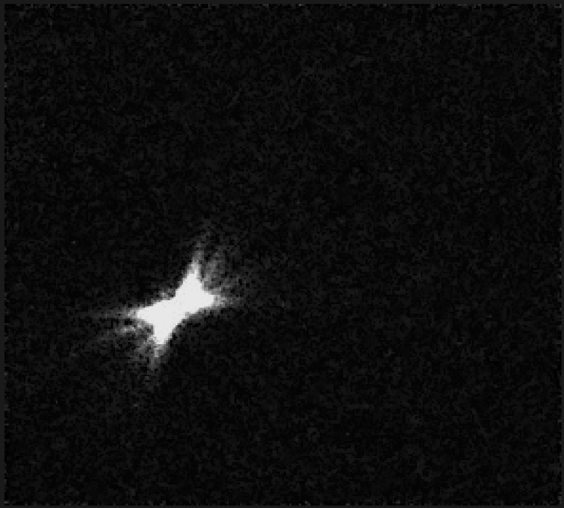


The secondary is to the upper left of the primary in the mask image.

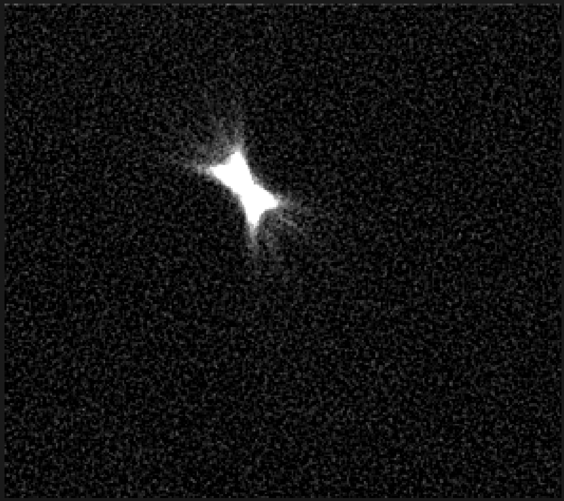
Is it real?

We took another image with the mask rotated about 90°. The rotated mask shows no hint of a secondary:

Original orientation:



Rotated:



Conclusions

- Detection of extrasolar terrestrial planets orbiting nearby stars is technically very difficult but may well be practical within the foreseeable future.

- A space-based telescope with an elliptical mirror and a shaped aperture provides the contrast needed to detect and perhaps characterize such planets.

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