Space Mission Concepts
To Image Earth-Like Planets In Habitable Zones

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2014 January 31

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http://www.princeton.edu/~rvdb
Are We Alone?

What Are The Odds?
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What Are The Odds?

This is Earth
Indirect Detection Methods

About 400 planets found so far
Wobble Methods

Radial Velocity.
For edge-on systems.
Measure periodic doppler shift.

Astrometry.
Best for face-on systems.
Measure circular wobble against background stars.
First Discovery: 51 Pegasi b

- Mayor and Queloz (1995)
- Mag. 5.5 main sequence star
- Detected by radial velocity method
- Velocity difference: 70 m/s = 160 mph
- Period: 4.2 days
- Separation: 0.05 AU
- Angular separation: 0.0035 arcseconds
- Mass: > 0.47 MJ
- Hot Jupiter
Notable “Recent” Discoveries

**Gliese 581 c (Possibly Terrestrial)**
- Mag. 10.5 red dwarf
- Detected by **radial velocity** method
- Period: 13 days
- Separation: 0.07 AU
- Angular Separation: 0.012 arcseconds
- Mass: $\approx 5M_E$

**Gliese 581 e (Hot)**
- Same Mag. 10.5 red dwarf
- Detected by **radial velocity** method
- Period: 3.1 days
- Separation: 0.03 AU
- Angular Separation: 0.005 arcseconds
- Mass: $\approx 1.7M_E$
Transit Method

- HD209458b confirmed both via RV and transit.
- Period: 3.5 days
- Separation: 0.045 AU (0.001 arcsecs)
- Radius: $1.3R_J$
- Intensity Dip: $\sim 1.7\%$
- Venus Dip = 0.01%, Jupiter Dip: 1%
- *Kepler* and *Corot*

Venus Transit (R.J. Vanderbei)

HD209458

![Diagram of transit and occultation](image)
EXOPLANETS COMPARED

PLANETS ARE SHOWN to scale in silhouette against their stars as if seen in transit. The sun and its planets, Pluto, and some moons are shown for comparison. We can discover the sizes of extrasolar planets by noting the fraction of their star’s light they block if they transit in front of it. Most planets discovered to date are very close to their stars and hence too hot to allow liquid water on their surface. Planet HD 209458 b is a hot gas-giant planet like Jupiter. Planet GJ 436 b is a hot Neptune-like planet. It’s hot because it is so close to its star, even though that star is a cool M-dwarf. CoRoT-7 b is the smallest transiting planet discovered so far—it’s diameter is only 1.7 times greater than Earth’s diameter. It is a rocky planet with a temperature of more than 1300K.
• *Space Interferometry Mission (SIM)*

• Wobbles as small as 0.000001 arcsecs (the thickness of a nickel viewed from the distance of the moon).

• Mission Cancelled
Direct Detection
Fomalhaut (First Detection via Direct Imaging)

Mag. 1.2, Distance 25 ly, Imaged by HST, Period: 872 years,

![Fomalhaut](image_link)
Why Earthlike in Habitable Zone is Hard

- **Bright Star/Faint Planet:** In visible light, our Sun is $10^{10}$ times brighter than Earth. That’s 25 mags.

- **Close to Each Other:** A planet at 1 AU from a star at 10 parsecs can appear at most 0.1 arcseconds in separation.

- **Far from Us:** There are less than 100 Sun-like stars within 10 parsecs.
Can Ground-Based Telescopes Do It?

- Atmospheric distortion limits *resolution* to about 1 arcsec. Note: Resolution refers to equally bright objects. If one is much brighter than the other, then it is more difficult.
- Segmented optics limits contrast
- Current adaptive optics not good enough

No they can’t (at least not yet)!
Can Hubble Do It?

No it can’t!

The problem is diffraction

Would have to be $1000 \times$ bigger (in each dimension!)
Telescope w/ Unobstructed Aperture

Doesn’t Work! Requires an aperture measured in kilometers to mitigate diffraction effects.
Three Classes of Solutions

- **Nulling Interferometers**
- Internal Coronagraphs
- External Occulters
Assessment: It’s hard—let the Europeans build it.
Three Classes of Solutions

- Nulling Interferometers
- Internal Coronagraphs
- External Occulters
Types of Coronagraphs (TPF-C)

- Hybrid Lyot *
- Apodized Pupils
- Shaped Pupils *
- Pupil Mapping (PIAA)
- Vector Vortex
- Phase Masks
- Visible Nuller
- Hybrids
Apodized Pupil Coronagraph

Diffraction Control via Tinting the Pupil

The abrupt edge of the telescope’s “mirror” causes the bright diffraction rings.

Solution: Use tinted glass to ease the transition from transparent to opaque.
Some of the Math

The image-plane electric field $E()$ produced by an on-axis plane wave (i.e., starlight) and an apodized (i.e., tinted) aperture defined by an apodization function $A()$ is given by the Fourier transform:

$$E(\xi, \zeta) = \iint e^{i(x\xi + y\zeta)} A(x,y) dy dx$$

$$E(\rho) = 2\pi \int_0^{1/2} J_0(r\rho) A(r) r dr,$$

where $J_0$ denotes the 0-th order Bessel function of the first kind.

**NOTE:** The electric field depends linearly on the apodization function.

The intensity is the square of the electric field.

The unitless pupil-plane “length” $r$ is given as a multiple of the aperture $D$.

The unitless image-plane “length” $\rho$ is given as a multiple of focal-length times wavelength over aperture ($f\lambda/D$) or, equivalently, as an angular measure on the sky, in which case it is a multiple of just $\lambda/D$. (Example: $\lambda = 0.5\mu m$ and $D = 10m$ implies $\lambda/D = 10\text{mas}$.)
Optimization

Find *apodization* function $A()$ that solves:

\[
\begin{align*}
\text{maximize} & \quad \int_0^{1/2} A(r)2\pi r\,dr \\
\text{subject to} & \quad -10^{-5}E(0) \leq E(\rho) \leq 10^{-5}E(0), \quad \rho_{iwa} \leq \rho \leq \rho_{owa}, \\
& \quad 0 \leq A(r) \leq 1, \quad 0 \leq r \leq 1/2, \\
& \quad -50 \leq A''(r) \leq 50, \quad 0 \leq r \leq 1/2
\end{align*}
\]

An infinite dimensional *linear programming* problem.
Pupil with “Optimal” Tinting

Mirror with Softened Edge

Image of Star

Mathematically Perfect... But Unmanufacturable!
Shaped Pupil Coronagraph

20 Petal mask

Image plane (20 petals)

Image plane (150 petals)

Still excellent, but still unmanufacturable.
Ripple3 Mask

Designed for an elliptical $4 \times 8$ meter primary.

$\rho_{iwa} = 4$

Throughput $= 30\%$

Note: throughput measured relative to ellipse

$11\%$ central obstr.

Easy to make

Only a few rotations
All above methods require optics of extraordinary quality: 1/10,000 wave precision.
Fig. 1.— Pupil mapping via a pair of properly figured lenses. Light travels from top to bottom.

Fig. 2.— (Left.) An amplitude profile providing contrast of $10^{-10}$ at tight inner working angles. (Right.) The corresponding on-axis point spread function.
Pure PIAA Doesn’t Work (Diffraction Analysis)
Hybrid PIAA/Apodized-Pupil

Fig. 4.— Analysis of an apodized pupil mapping system using the S-Huygens approximation with $z = 1.5D$ and $n = 1.5$.

Upper-left plot shows in red the target high-contrast amplitude profile and in blue the amplitude profile computed using the S-Huygens approximation through the apodized pupil mapping system. The other two curves depict the pre- and post-apodizers.

Upper-right plot shows the lens profiles, red for the first lens and blue for the second. The lens profiles $h$ and $\tilde{h}$ were computed using a $5,000,000$ point discretization.

Lower-left plot shows in blue the phase map computed using the S-Huygens propagation with a $5,000,000$ point discretization. This blue curve exhibits high-frequency oscillations that are smoothed before computing the post-apodizer to help with chromaticity.

Lower-right plot shows the PSF computed at three different wavelengths; the design value, 30% above that value, and 30% below it.
What About Imperfect Optics?

So far, we have assumed perfect optics.

Manufacturing errors are inevitable. They could be partially corrected using deformable mirrors (DMs) and a wavefront sensing system.

Thermal changes, vibrations, and possibly other effects will necessitate a dynamic wavefront control system.

Can we correct wavefront errors enough to achieve 25 magnitudes of contrast?

Neptune

Neptune w/ Adaptive Optics
Jeremy Kasdin tinkers with the laser.
More postcards from the edge...
Three Classes of Solutions

- Nulling Interferometers
- Internal Coronagraphs
- *External Occulters*
Nature's Coronagraph

Use an external occulter to block the light.
The fundamental size and separation for a starshade are LARGE.

Shadow size given by $R$

Inner Working Angle given by:

$$\tan \theta = \frac{R}{z}$$

For $D = 4 \text{ m}$, $R = 3 \text{ m}$, and IWA = 75 mas, $z \sim 10,000 \text{ km}$
Poisson didn’t believe the wave theory of light. He pointed out that light falling on a circular object would have a bright spot at the center of its shadow.

Arago did the experiment.

Poisson was wrong.
A Fun Experiment
Plain External Occulter (Doesn’t Work!)

Circular Occulter

Simulated star/planet image

Poisson’s Spot!

Shadow isn’t dark enough
Shaped Occulter

![Diagram of a shaped occulter](image1)

![Graph showing x and y coordinates](image2)

![Simulated visualization of the occulter](image3)

![Detailed close-up of the occulter](image4)
Space-based Occulter (TPF-O)

Telescope Aperture: 4m, Occulter Diameter: 50m, Occulter Distance: 72,000km
Starshade Stowage and Deployment
A Real Petal...
...And How It Furls
Our Solar System From Fomalhaut
Ground-Based Possibilities

- Atmospheric seeing limits resolution to about 1 arcsec.
- Large aperture with adaptive optics.
- Interferometry.
Which Space-Based Observatory Seems Easiest To Build...

**Interferometer.** A cluster of two to four multi-meter sized infrared telescopes flying in formation at separations on the order of 100 meters with subwavelength precision so that the central star can be attenuated by 25 magnitudes by destructive interference.

**Coronagraph.** A four to eight meter off-axis telescope with built-in diffraction control scheme and active adaptive optics to maintain unprecedented wavefront quality \((1/10,000\text{-th wave})\) over the course of very long exposures (light throughput of the diffraction control system is only about 10%).

**Occulter.** A four meter diffraction limited telescope and a specially configured 50 meter tip-to-tip occulter “flying” 72,000 km in front of the telescope with station-keeping to within a ±1 meter tolerance over the course of a multihour exposure.

REMINDER: We landed humans on the moon and brought them safely home again.
AFTA Satellite

Repurposed NRO Spy Satellite

Similar to Hubble.
Aperture: 2.4 meters.
Central Obstruction and Spiders.