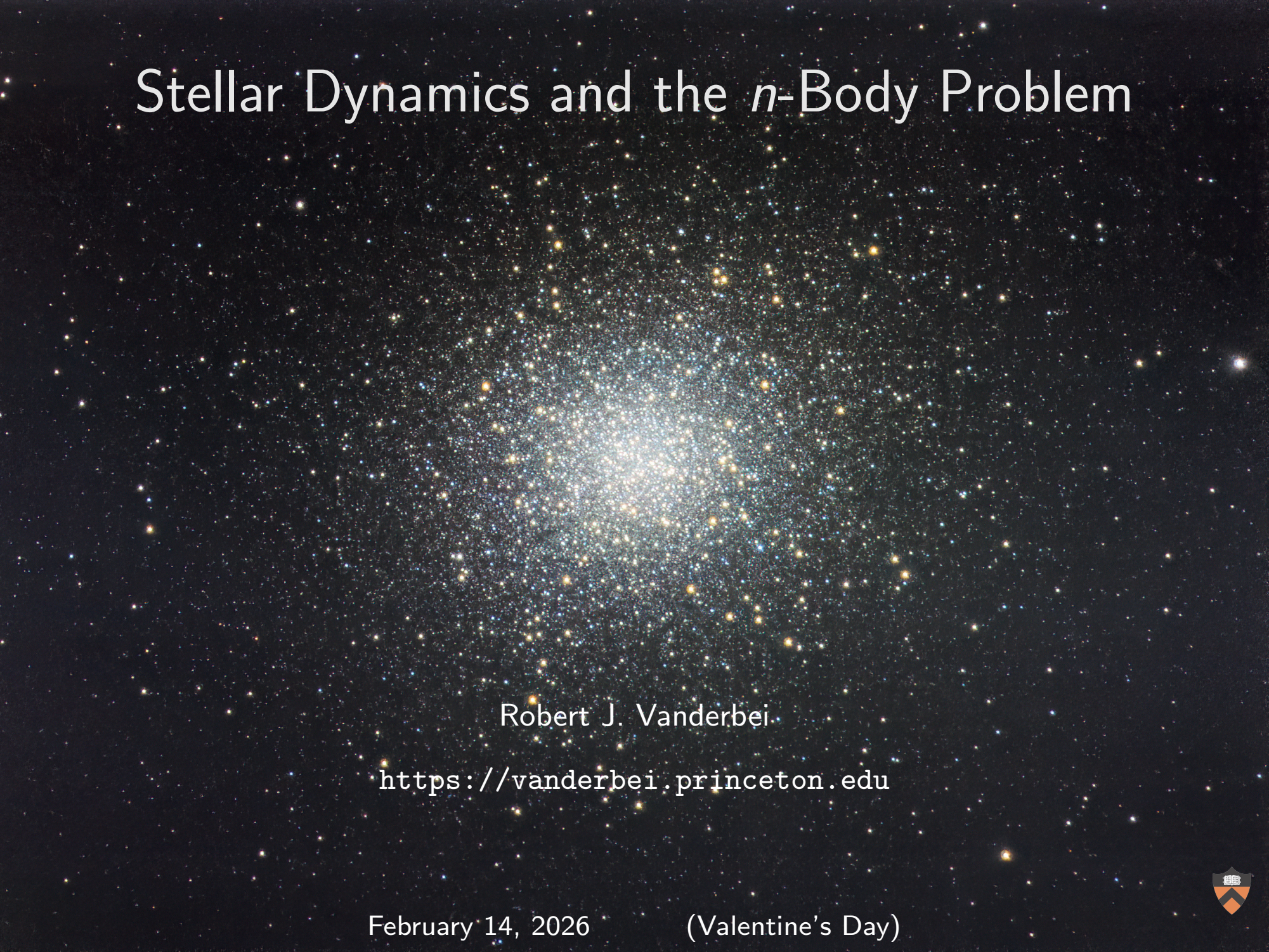
Stellar Dynamics and the n-Body Problem



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This Talk is all about Dynamics

- It's very difficult to show dynamic animations in a pdf file.
- For that reason, this pdf file just has links to online html files that show the dynamics.
- Get your mouse ready to do lots of clicking.

Stellar Dynamics

- I downloaded from the Hipparcos database the Position (RA and Dec), Proper motion (RA and Dec), Radial Velocity, Magnitude and Color Index of all stars brighter than Magnitude 10.

- I was hoping to use Gaia data but Gaia doesn't have this data for most stars that are brighter than Magnitude 5.

- Using this Hipparcos data, I was able to show how star positions change dramatically over thousands of years.

- Here it is...

Year: 3351

Resume Now

FOV: 90°

Max magnitude: 10.0

Hide Constellation Lines Hide Star Labels

Stars Moving Over Time

Use the sliders to change year & FOV. Click anywhere in the view to re-center.

Click here to see a stereoscopic 3D rendition.

To change the view point, either type an Object Name (M, NGC, IC, or HD): Barnards Star

or give a Direction to Look: Select

or enter RA and Dec coordinates

RA: 269.450, Dec: 4.693

or just click somewhere.

Note: Barnard's Star is being shown as magnitude 1 when in reality it is magnitude 9.5.

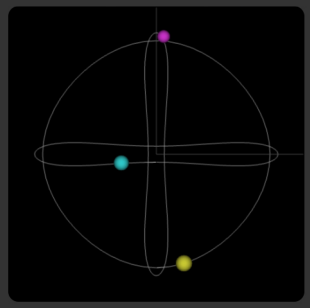
Stellar Dynamics Closer to Home

- The link on the following page takes you to a webpage with lots of links to webpages that show how dynamics affects our Solar System.
- Lots of things are shown including Lagrange Points, Star Dust, Coorbtial Asteroids, and Horseshoe orbits.
- Here it is...

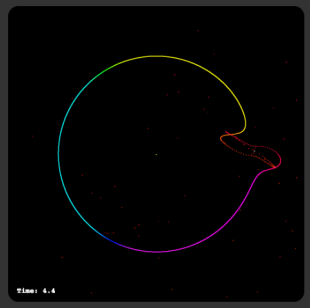
Star, Planet, Moon Dynamics

← Homepage

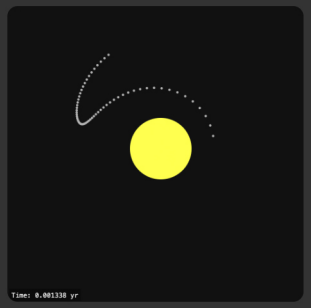
Click on the icons to see the dynamics



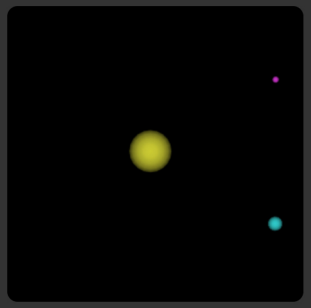
Ducati: Star/Planet/Moon



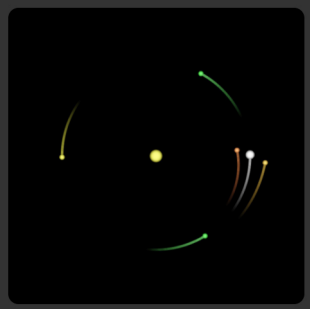
L4/L5 Accumulation of Star Dust



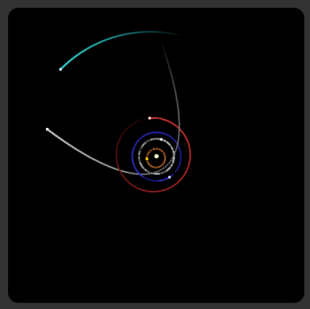
Saturn and Janus sized moons



Lagrange 3-Body

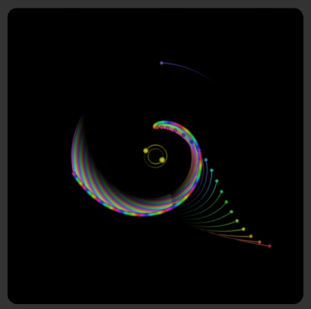


Lagrange L1 to L5

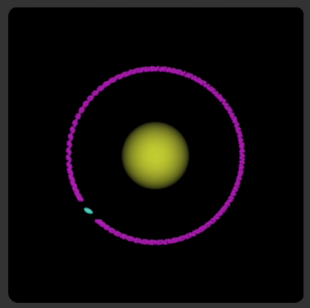


Solar System

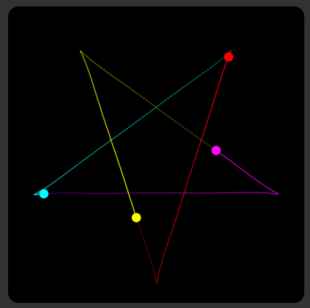
Earth Coorbital Asteroids



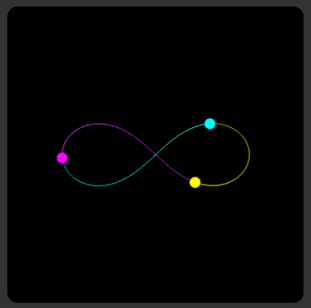
CircumBinary Planets



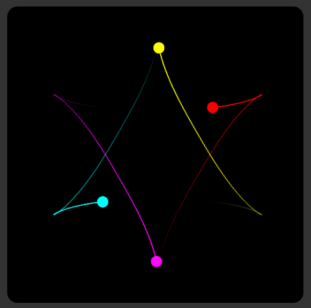
Horseshoe Orbit
Janus, Epimetheus and Saturn



Ouyang-Xie Examples



Ouyang-Xie Figure 8



New Periodic Solutions

Periodic Solutions to the n-Body Problem

- On the following pages you'll get to see lots of new solutions to the n-Body problem where n is mostly either 3 or 4.
- A few of these solutions were discovered by me.
- Most of them were discovered by Li and Liao.
- For those of you who like Physics and Mathematics, I can say that these periodic orbits were all found by searching for local minima to the *Action Function*:

$$A(p_1, ..., p_n) = \int_0^T \left(\frac{1}{2} \sum_{i=1}^{n} m_i \|\dot{p}_i(t)\|^2 + \sum_{1 \le i < j \le n} \frac{m_i m_j}{\|p_i(t) - p_j(t)\|} \right) dt$$

  where $p_i(t)$ is the position of body i at time t, m_i is the mass of body i and T is the orbital period.
- PS. All of the orbits are truly *periodic*. But, only some of them are *stable*.
- Here they are...

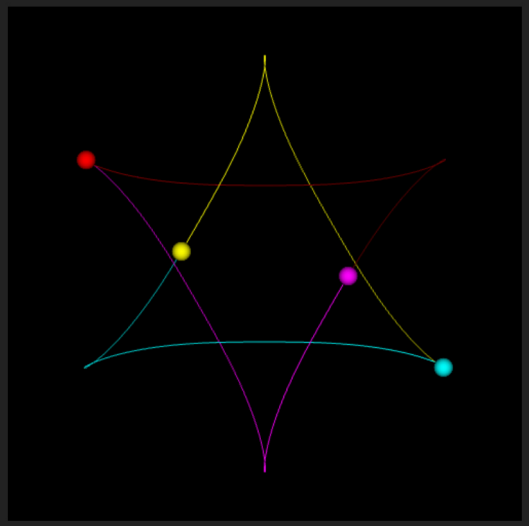
Periodic Planar Solutions to the n-Body Problem

← Homepage

Use the pull-down `Select an orbit' menu or click on the icons to check out other new orbits.

The first in the list is *Star of David*. I discovered it on June 24, 2013.

Click here to see in 3D a few hundred stable periodic orbits to the 3-body problem.



Time = 1.35

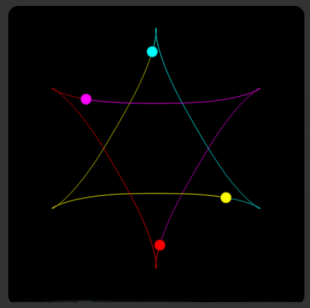
Reset | Pause/Run | Select an orbit: Star of David

Delay between frames: 30 ms. Redraw dt: 6e-2. Integration dt: 1e-4.

Integration Method: Leapfrog. Toggle Counter-rotation To check stability... Perturb Orbits

Drag mouse to rotate 3D model. Hold *shift* key to zoom in and out.

The IAS15 integrator is a new high-precision integrator by Rein and Spiegel.



Star of David

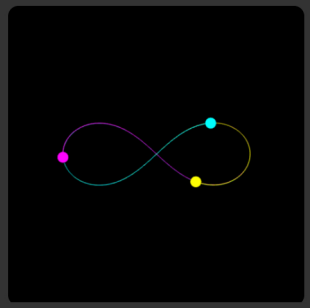


Figure Eight 3-Body

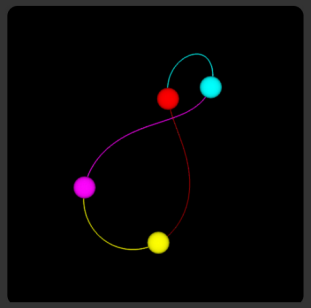
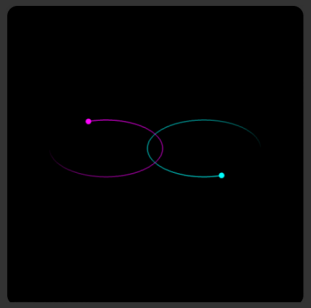
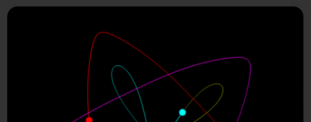


Figure Eight 4-Body



Double Ellipse



Periodic Solutions to the n-Body Problem
Discovered by XiaoMing Li and ShiJun Liao

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Switch to 2D (single-eye) view

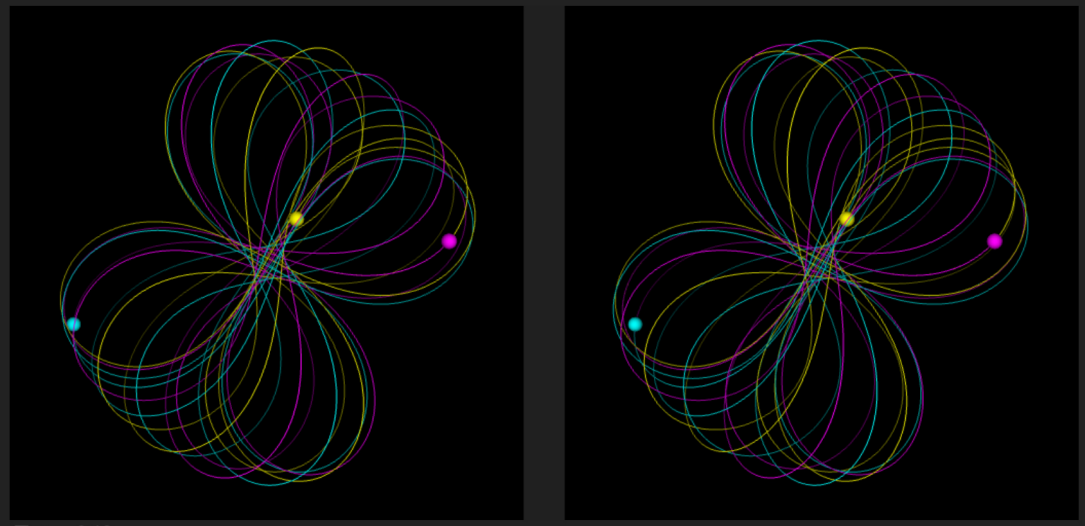
References: https://numericaltank.sjtu.edu.cn/three-body/2017-SCPMA-3body.pdf
https://arxiv.org/pdf/2508.08568

Point your left eye at the left image and your right eye at the right image and enjoy the 3D stereoscopic view.

Click and drag your mouse to change your viewing posiition.

Here are 185 equal-mass (mostly) stable periodic orbits: -- Select an orbit --

O_{1497}

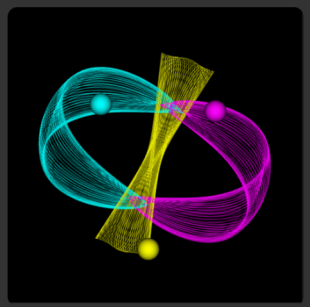


Time = 0.19

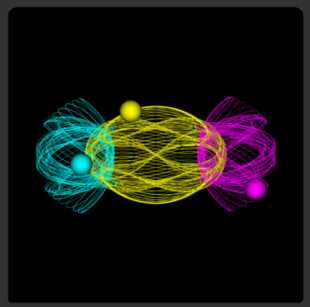
Reset Pause/Run

Delay between frames: 30 ms. Redraw dt: 6e-2. Integration dt: 1e-4. Integration Method: IAS15.

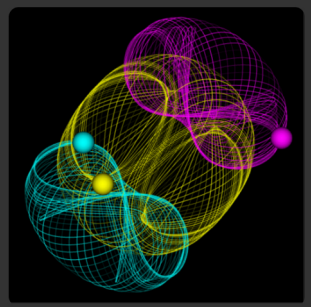
Drag mouse to rotate 3D model. Hold *shift* key to zoom in and out.
The IAS15 integrator is a new high-precision integrator by Rein and Spiegel.



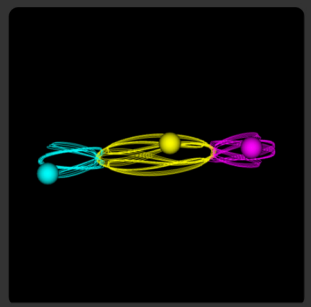
$I.A^{i.c.}_{100}$



$I.A^{i.c.}_{115}$



$I.B^{i.c.}_{102}$



$I.B^{i.c.}_{155}$ Unstable