Extinguishing Poisson’s Spot with Linear Programming

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2008 May 13

SIAG/OPT 2008
Boston, MA

http://www.princeton.edu/~rvdb
Are We Alone?
Indirect Detection Methods

Almost 300 planets found so far
Wobble Methods

Radial Velocity.
  For edge-on systems.
  Measure periodic doppler shift.

Astrometry.
  Best for face-on systems.
  Measure circular wobble against background stars.
Transit Method

- HD209458b confirmed both via RV and transit.
- Period: 3.5 days
- Separation: 0.045 AU (0.001 arcsecs)
- Radius: $1.3R_J$
- Intensity Dip: $\sim 1.7\%$
- Venus Dip = 0.01%, Jupiter Dip: 1%

[Image of Venus Transit]
Direct Detection
Why It’s Hard

- **Bright Star/Faint Planet:** In visible light, our Sun is $10^{10}$ times brighter than Earth. That’s 25 mags.

- **Close to Each Other:** A planet at 1 AU from a star at 10 parsecs can appear at most 0.1 arcseconds in separation.

- **Far from Us:** There are less than 100 Sun-like stars within 10 parsecs.
Telescope w/ Unobstructed Aperture

Doesn’t Work! Requires an aperture measured in kilometers to mitigate diffraction effects.
Space-based Occulter (TPF-O)

Telescope Aperture: 4m, Occulter Diameter: 50m, Occulter Distance: 72,000km
Plain External Occulter (Doesn’t Work!)

Shadow ⇒

Circular Occulter ⇓

Note bright spot at center (Poisson’s spot)

Telescope Image

← Shadow (Log Stretch)
Shaped Occulter—Eliminates Poisson’s Spot

Shadow ⟹

Shaped Occulter

⇓

Bright spot is gone

Telecope image shows planet

⇒ Shadow is dark
(Log Stretch)
Apodized Occulters

• The problem is *diffraction*.

• Abrupt edges create unwanted diffraction.

• **Solution:** Soften the edges with a partially transmitting material—an *apodizer*.

• Let $A(r, \theta)$ denote *attenuation* at location $(r, \theta)$ on the occulter.

• The *intensity* of the downstream light is given by the *square of the magnitude of the electric field* $E(\rho, \phi)$.

• *Babinet’s principle* plus *Fresnel propagation* gives a formula for the downstream electric field:

$$E(\rho, \phi) = 1 - \frac{1}{i\lambda z} \int_{0}^{\infty} \int_{0}^{2\pi} e^{\frac{i\pi}{\lambda z} (r^2 + \rho^2 - 2r\rho \cos(\theta - \phi))} A(r, \theta) r \, d\theta \, dr.$$

where

- $z$ is distance “downstream” and
- $\lambda$ is wavelength of light.
Attenuation Profile Optimization

\[
\begin{align*}
\text{minimize} & \quad \gamma \\
\text{subject to} & \quad -\gamma \leq \Re(E(\rho)) \leq \gamma & \text{for } \rho \in \mathcal{R}, \lambda \in \mathcal{L} \\
& \quad -\gamma \leq \Im(E(\rho)) \leq \gamma & \text{for } \rho \in \mathcal{R}, \lambda \in \mathcal{L} \\
& \quad A'(r) \leq 0 & \text{for } 0 \leq r \leq R \\
& \quad -d \leq A''(r) \leq d & \text{for } 0 \leq r \leq R
\end{align*}
\]

Specific choice:

\[R = 25, \quad d = 0.04, \quad \mathcal{R} = [0, 3], \quad \mathcal{L} = [0.4, 1.1] \times 10^{-6}\]

where all metric quantities are in meters.

An infinite dimensional linear programming problem. Discretize:

- \([0, R]\) into 5000 evenly space points.
- \(\mathcal{R}\) into 150 evenly spaced points.
- \(\mathcal{L}\) into increments of \(0.1 \times 10^{-6}\).
Petal-Shaped Occulters

- From Jacobi-Anger expansion we get:

\[
E(\rho, \phi) = 1 - \frac{2\pi}{i\lambda z} \int_0^R e^{\frac{i\pi}{\lambda z}(r^2 + \rho^2)} J_0 \left( \frac{2\pi r \rho}{\lambda z} \right) A(r) dr \\
- \sum_{k=1}^{\infty} \frac{2\pi (-1)^k}{i\lambda z} \left( \int_0^R e^{\frac{i\pi}{\lambda z}(r^2 + \rho^2)} J_k \left( \frac{2\pi r \rho}{\lambda z} \right) \frac{\sin(\pi k A(r))}{\pi k} dr \right) \\
\times \left( 2 \cos(kN(\phi - \frac{\pi}{2})) \right)
\]

where \( N \) is the number of petals.

- For small \( \rho \), truncated summation well-approximates full sum.

- Truncated after 10 terms.

- \( \lambda \in [0.4, 1.1] \) microns.

- \( z = 72,000 \) km, \( R = 25 \) m.

- In angular terms, \( R/z = 73 \) mas.