Deep Shadow Occulters

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Web’s Hypergaussian Radial Amplitude Profile

\[ A(r) = \begin{cases} 
1 & \text{for } r < a \\
 e^{-((r-a)/b)^n} & \text{for } a \leq r < c \\
0 & \text{for } c \leq r 
\end{cases} \]

where \( a = b = 6.25 \), \( c = 20 \), \( n = 6 \) and distance from JWST is 18,000 km.
Okay at visible wavelengths, but degrades quickly as wavelength increases.
Planet at tip of flower. Planet intensity $10^{-10}$ times intensity of star at $\lambda = 0.8\mu$. 
Optimized Radial Amplitude Profile

Distance from JWST: 18,000 km
$10^{-10.5}$ at $\lambda = 0.8\mu$; $10^{-9.5}$ at $\lambda = 1.2\mu$; $10^{-8.5}$ at $\lambda = 1.6\mu$;

$10^{-7.5}$ at $\lambda = 2.0\mu$; $10^{-6.5}$ at $\lambda = 2.4\mu$;
Performance at Other Wavelengths
Planet at tip of flower. Planet intensity $10^{-10}$ times intensity of star at $\lambda = 0.8 \mu$. 
minimize \( \int_0^\infty A(r) r \, dr \)

subject to

\[ -10^{-c_j} \leq \Re(E_j(\rho)) \leq 10^{-c_j}, \quad 0 \leq \rho \leq 4, j = 1, 2, \ldots, 5 \]
\[ -10^{-c_j} \leq \Im(E_j(\rho)) \leq 10^{-c_j}, \quad 0 \leq \rho \leq 4, j = 1, 2, \ldots, 5 \]
\[ A(r) = 1, \quad 0 \leq r \leq 6.25 \]
\[ A'(r) \leq 0, \quad 6.25 \leq r \leq 20 \]
\[ A(r) = 0, \quad 20 \leq r \leq \infty \]
\[ -\gamma \leq A''(r) \leq \gamma, \quad 6.25 \leq r \leq 20, \]

where

- \( \gamma \) is a smoothness parameter,
- \( c_j \) specifies the level of contrast at wavelength \( \lambda_j \),
- \( z = 18,000 \) km, and
- by Babinet’s principle

\[
E_j(\rho) = 1 - \frac{2\pi}{i\lambda_j z} \int_0^\infty e^{\frac{i\pi}{\lambda_j z}(r^2+\rho^2)} A(r) J_0 \left( \frac{2\pi r \rho}{\lambda_j z} \right) r \, dr
\]