Sizing Up The Universe

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UACNJ at Jenny Jump

http://www.princeton.edu/~rvdb
The Whole Sky

Whole sky showing Milky Way
Andromeda and Moon
Sirius

HD209458

Buzz Aldrin’s Footprint

Proxima Centauri

Alpha Centauri

0.002 arcseconds
Measuring Distances...
Cleverness Required
The sun and the moon are the same apparent size in the sky, each about half a degree across. Never is this more apparent than when the moon crosses exactly between Earth and the sun to cause a solar eclipse. The moon's orbit around Earth is not a perfect circle, so its distance from Earth changes a little during its orbit each month. When it is farther away, it looks smaller, and when it is closer, it looks bigger.

If the moon crosses in front of the sun while at its greatest distance from Earth, it causes an annular solar eclipse, as shown in the photograph opposite. Being a bit farther away than usual, the moon is slightly smaller in apparent size than the sun. Although it almost completely blots out the sun, it leaves a bright ring, or annulus, visible around the edge. This annular eclipse allows us to gauge the relative angular sizes of the moon and the sun—or how big they appear to an observer on Earth. If the moon passes in front of the sun when the moon is closer to Earth, its image is larger and can completely cover the bright surface of the sun; we refer to this as a total solar eclipse. During the few minutes when the moon is covering the sun, one can see the sun's faint outer atmosphere, its corona.

Because the moon blots out the sun when it passes in front of it, the ancient Greeks knew that the moon must be closer to us than the sun is. If the sun is about the same apparent size as the moon in the sky but is farther away, then it must actually be bigger than the moon.

Apparent, or angular, sizes are important. The angular size of the sun plays a crucial role in determining the temperature of Earth. If the sun were just twice its current angular diameter (say, by being only half as far away), it would take up four times as much area in the sky, and it would deliver four times as much sunlight to Earth. That would be like having four suns up in the sky! Earth would become a lot hotter.

With Earth's current reflectivity and atmospheric greenhouse effects, if the sun were twice as large in angular diameter in the sky, the Earth's average temperature would increase by 126°C (227°F). That's above the boiling point of water: The oceans would boil. If the sun were half its current apparent size in the sky (by making it twice as far away), the temperature of the Earth would drop by 84°C (151°F), and the oceans would freeze. So it's no accident that the sun is the size it appears: If it were much larger or smaller in the sky, we wouldn't be here to appreciate it.

Along these same lines, since the angular sizes of the sun and the moon are the same, their tidal effects on Earth would be equal if their internal densities were the same. However, the moon, which is made up of rock, is about 2.4 times as dense as the sun, which is made up of gas. Thus, tides produced by the moon are about 2.4 times larger than those produced by the sun. Ocean tides follow the moon but are modulated in amplitude according to the sun's position relative to the moon.

Sir Isaac Newton (1642–1727) figured out that ocean tides are caused by the gravitational effect of the moon. Because tides produced by the moon are stronger than those produced by the sun, he deduced correctly that the moon is denser than the sun. Newton's theory of gravity tells us that if the moon were twice as large in apparent size—by being twice as close—the ocean tides would be eight times as large, swamping coastal areas. Angular sizes are important indeed.
DiStaNcEs
mast last. Also, as one travels farther and farther south, new stars appear above the horizon that were not visible in the north. That would not occur if Earth were flat. Finally, during a lunar eclipse, when Earth passes directly between the sun and the moon, you can see Earth's shadow on the moon. Its outline is circular—always. No matter where the moon is relative to your observation point, the shadow of Earth always appears to be circular. The only shape that always casts a circular shadow is a sphere.

So people have known that Earth is a sphere since the time of Aristotle and before. However, they still assumed that Earth was at the center of the universe. Eudoxus of Cnidus (408–355 b.c.) thought the celestial sphere carrying the stars surrounded a stationary, spherical Earth at the center. The celestial sphere rotated on its axis once a day, causing the stars to rise and set. A transparent sphere nested inside the celestial sphere carried the sun on a 360-degree journey relative to the stars once a year. The moon and planets were supposed to move on a complicated additional set of nested spheres, with various rotations. Aristarchus of Samos (320–250 b.c.) disagreed—he said the celestial sphere does not move. Instead, it is Earth that rotates on its axis once a day. The stars rise and set, appearing to move, simply because Earth is turning. He was right!

The moon orbits Earth once a month, and Aristarchus figured out that it shines by the light it receives from the sun and moon both have the same angular diameters in the sky—one-half degree. That means that if they were at the same distance from Earth, they would be the same size. But since the sun is farther away, it is bigger. We now know that the sun is about 400 times farther away than the moon, and therefore 400 times as large as the moon.

We can figure out the physical size of an object if we know its angular size and its distance. We use the principles of Euclidean geometry. The circumference of a circle is $\pi$ times its diameter; $\pi$ is 3.14159265\ldots, a decimal number with an infinite number of digits following the decimal point. Many billions of digits of $\pi$ have been computed, with more being added all the time. The radius of a circle is half its diameter, so the length of the circumference is just $2\pi$ times its radius. The circumference of a circle is 360 degrees.

Mathematicians call the angle $\left(\frac{360\text{ degrees}}{2\pi}\right)$ one radian. It can be used as a unit of measure. To convert from degrees to radians, just multiply...
How Eratosthenes measured the size of the Earth

Eratosthenes (276–194 B.C.) found out. He worked in Alexandria, Egypt, where he was the chief librarian of the Library of Alexandria. He knew that at noon on the longest day of the year, June 21, the sun famously stood directly overhead in the town of Syene, south of him on the Nile River (at modern-day Aswan), because one could look down a deep well there at that time and see the reflection of the sun in the water. But he also knew that the sun did not stand directly overhead at noon on June 21 in Alexandria. An obelisk in Alexandria cast a shadow then. If the sun were directly overhead, it would cast no shadow. By measuring the height of the obelisk and the length of the shadow, he determined that the sun was 7.2 degrees off vertical at noon on June 21 in Alexandria.

The difference was the result of the curvature of Earth. Eratosthenes knew the sun was very far away, so light rays from the sun essentially traveled on parallel lines toward Earth. If Earth’s surface curved by 7.2 degrees between Syene and Alexandria, as it would if they were separated by 7.2 degrees of latitude on the globe, then that would cause the sun to be 7.2 degrees off vertical in Alexandria, as shown opposite. The angle between Alexandria and Syene as seen from the center of Earth should be equal to the amount by which the sun was off vertical in Alexandria (the two red arcs). These angles are equal if the rays from the distant sun are essentially parallel, according to a famous theorem in Euclidean geometry.

Because 7.2 degrees is one-fiftieth of a full circle (360 degrees), Eratosthenes reasoned that the distance between Syene and Alexandria should be one-fiftieth of the circumference of Earth. Now all he needed to do was send someone on a trip directly south from Alexandria to Syene to measure the distance between them. Multiply this number by 50, and he would have the circumference of Earth. He got a circumference of about 40,000 kilometers (25,000 miles) in today’s units, pretty close to the true value. When you divide this number by \( \pi \), you get the diameter of Earth: approximately 12,730 kilometers (7,910 miles). Multiply by 30, and you have the distance to the moon—about 382,000 kilometers (237,000 miles).

Eratosthenes’ method was important because it set the scale for distances measured later, which would often be measured first relative to the size of Earth.

Later, Posidonius (135–50 B.C.) repeated Eratosthenes’ work, using the star Canopus. He measured in degrees how far it rose above the southern horizon as seen from the Greek island of Rhodes, compared with Alexandria, which was farther south. For Earth’s circumference, Posidonius obtained a value of only about 29,000 kilometers (18,000 miles)—quite a bit too small. However, Posidonius was friends with Cicero and Pompey, and the influential second-century astronomer Ptolemy chose Posidonius’ value over Eratosthenes’.

That wrong answer of Posidonius affected the course of world history. People learned from Marco Polo’s travel accounts the approximate overland distance between Europe and China. Using Posidonius’s value for the circumference of Earth, Christopher Columbus figured the distance he would have to sail, traveling west from Europe, to circle the globe and arrive in China. With Posidonius’s smaller Earth, the trip looked short and favorable for a trade route—so he embarked.
How Aristarchus measured the size of the Moon
Measuring the distance to Venus

The distance between Earth and the sun is called the astronomical unit, or AU for short. How do we find it?

First, we find the relative sizes of the orbits of the planets. Venus's orbit is nearly circular, and as it and Earth both circle the sun at different rates, Venus keeps lapping Earth, oscillating from one side of the sun to the other as time progresses (opposite). The farthest Venus ever gets from the sun is 46 degrees, shown by its two limiting positions at the top and bottom. If you draw dots for the sun and Earth, you can draw with a protractor two lines at 46 degrees with respect to the sun, just like the diagonal lines shown in the figure. Now draw a circle around the sun with a compass that just touches those two diagonal lines. That's the circular orbit of Venus drawn to the same scale as the distance between Earth and the sun. If Venus wanders to a maximum elongation of 46 degrees with respect to the sun, the radius of Venus's orbit is 0.72 AU.

After Venus reaches greatest eastern elongation from the sun as the evening star, the angle between it and the sun begins to drop below 46 degrees. We can draw the angle we observe and extend a line from Earth at that angle. Where it intersects the orbit of Venus, that's Venus's position. Measuring the length of this line segment and comparing it with the distance between Earth and the sun allows us to calculate the distance to the sun in terms of the distance between us and Venus at a given time. We can find the distance between us and Venus using the parallax effect by observing from two locations as shown below.

For example, occasionally Venus passes in front of the bright star Regulus (an occultation). Suppose we observe England to Regulus and South Africa to Regulus to the sun from the earth (opposite). Seen from South Africa (below), Venus passes directly in front of Regulus; from England, it misses. We can measure the angle at England—it's equal to the angle at Venus. That tells us the angular separation of England and South Africa (a known distance apart) as seen from Venus, and therefore the distance to Venus.
Measuring the distance to the Sun

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Parallax: Distance to the Stars

So Why Didn’t People Believe Aristotle?

Because Aristotle had a killer argument for why Earth did not move. If Earth circled the sun, the stars should show a parallax effect—and this was not seen. As Earth circled the sun, Earth’s position relative to the stars should oscillate, causing the stars’ positions to oscillate once a year in the sky. This is explained in the figure opposite. The true situation is as shown at the top—just as Aristarchus envisioned it. Earth circles the sun once a year. Assume the stars and the sun remain at fixed positions. How does it look from Earth? We are riding on Earth, so it looks to us like Earth is not moving.

It looks to us like the sun moves in a small circle of radius 1 AU around Earth once a year (that’s why it circles the celestial sphere once a year). The stars do not move relative to the sun, so as seen from Earth, stars must, like the sun, also seem to move in 1 AU circles over the course of a year. We should be able to see the stars trace these circles in the sky every year. These parallax circles represent the reflex motion of the stars relative to Earth produced by the motion of Earth as it circles the sun, creating changing viewing angles during the year to those stars (top right). If the distance from Earth to the sun is 1 AU, then the radius of all these parallax circles would also be 1 AU. The angular radius of the parallax circle depends on the distance to the star. A nearby star has a larger angular oscillation in the sky as seen from Earth than a distant star (bottom right).

If we look at a constellation, the nearby stars should oscillate more during the year than the distant stars. So the positions of nearby stars should shift during the year relative to more distant stars. The ancients thought that the stars were close enough that these oscillations should have been visible to the naked eye. But none were seen. Aristotle thought that proved Earth didn’t move.

Aristarchus proposed an answer—no parallax effects were seen because the stars were infinitely far away. Parallax effects get smaller the farther away the stars are. Put the stars twice as far away, and the parallax effects become half as large. Put them infinitely far away, and the parallax effects disappear entirely. It was almost the right answer.

In 1453 Nicolaus Copernicus (1473–1543) published a sun-centered model based on Aristarchus’s work. In it he was able to explain in a simple manner the main motions seen in the solar system. Mercury and Venus oscillate back and forth ahead of and behind the sun as the sun circles the sky once a year. Copernicus said this is because they, like Earth, orbit the sun but are closer to the sun than Earth is. Before Copernicus, people had explained this motion with epicycles: The planet was supposed to circle a point that itself circled Earth. The big circle carrying the point was called the deferent, and the small circle around that point was called the epicycle. Venus and Mercury had large deferent circles exactly synchronized with the sun. Their epicycles produced their oscillations around the sun. The outer planets (Mars, Jupiter, Saturn) had big deferent circles that traced their slow orbits around the sky and epicycles with periods of one year each, which in reality showed the reflex (parallax) motion relative to Earth caused by Earth’s movement around the sun.
In 1912 Henrietta Leavitt (1868–1921) found a relation between the brightness of Cepheid variable stars and their period of variability. Over the course of 50 years, the brightness of these stars varied by a factor of 100, observed in nearby globular star clusters. By comparing the brightness of main sequence stars in the same cluster, she could determine the distance to the cluster. This allowed her to construct a distance scale to go from the brightness of main sequence stars to the brightness of supernovae, and, for very distant galaxies, to Cepheid variables. By 1923, Edwin Hubble (1889–1953) had identified Cepheid variables in the Andromeda galaxy (M31) with the 100-inch-diameter telescope on Mount Wilson and resolved M31 into stars. He found the ensemble of globular clusters in M31 is 2.5 million light-years away. By extrapolating the proper motions of globular clusters, he estimated the recession velocity of the Andromeda galaxy (M31). Hubble found, the larger their distance, the larger their redshifts were. So galaxies are moving away from us. Moreover, the farther away they are, the faster they are moving. The universe is expanding! By 1931 Hubble had found a distant galaxy receding at a speed of almost 20,000 kilometers per second. Today we can measure the velocities of distant galaxies using their redshifts. Galaxies showing a redshift are moving away from us. On average, for every million parsecs (a megaparsec) farther away a galaxy is from us, it will be going 71 kilometers per second faster. A galaxy that is 10 megaparsecs away from us will have a recessional velocity of 710 kilometers per second, and one that is 100 megaparsecs away from us will have a recessional velocity of 7,100 kilometers per second. The galaxies are moving apart as the universe expands.
Supernovae: Distance to Remote Galaxies

M51 (May 9, 2005)

M51 (July 10, 2005)

Arrows point to supernova
Looking Out is Looking Back

13.7 Billion Years Ago

The cosmic microwave background is the most distant thing we can see in the universe. This radiation comes to us from a distance of about 13.7 billion light-years.
The Universe in Ten Steps
Actual Size

Buzz Aldrin's Footprint on the Moon
1:1 Thousand

Asteroid Itokawa

Astronaut McCandless

Space Shuttle

Hubble Space Telescope

International Space Station
Asteroids, comets, and moons

1:1 Million

Comet Wild 2
Halley’s Comet
951 Gaspra
Phobos
Deimos
Enceladus
Neutron Star
Cygnus X-1 Black Hole

50.8 km

TO INFINITY AND BEYOND: 200

Cygnus X-1 Black Hole
to infinity and beyond

1:1 Trillion

Black hole at
galactic center

Comet Hyakutake

Size of previous step

Gliese Solar System

Orbit of d

Betelgeuse

B's orbit

Proxima Centauri

Rigel

Mar's orbit

Vega

Earth's orbit

Tau Ceti

Venus's orbit

Alpha Centauri

Mercury's orbit

Sun

HD 209458

1:1 Trillion

Stars
1:1 Quadrillion

The Solar System

Betelgeuse
Vega dust disk

(Orbits, from small to large)
Jupiter
Saturn
Uranus
Neptune
Halley’s Comet
Pluto
Eris
Sedna

Black hole in M87
Fomalhaut-b’s orbit

Size of previous step

50 billion km
Globular Clusters & Nebulae

1:1 Quintillion

- Sun
- Alpha Centauri
- Orion
- Dumbbell
- Eskimo
- Ring
- Crab
1:1 Sextillion

Andromeda Galaxy (M31)

Size of previous step

G64

G76

5,370 ly
1:1 Septillion

Galaxy Clusters

Perseus Cluster

Cosmic Microwave Background

Coma Cluster

Bullet Cluster

Local Group

Milky Way

M31

M33
Step Ten

ONE LAST JUMP by a factor of a thousand, and we can see the extent of the entire visible universe—everything we can see. We are looking here at the equatorial slice of the Sloan Survey containing 126,594 galaxies and quasars. It is a cross-sectional slice of the universe extending outward from Earth's Equator. Earth is in the center of the picture. Galaxies are shown as green dots, and quasars as orange dots. The two large, black regions are zones of avoidance, where our galaxy blocks the distant view. The scale shows the look-back-time distance in billions of light-years.

When we look out in space, because of the finite velocity of light, we look back in time. A galaxy five billion light-years away we see as it was five billion years ago. We can see out to a radius of just 13.7 billion light-years in any direction, because the universe began in a big bang explosion 13.7 billion years ago. The farthest thing we can see is the cosmic microwave background radiation left over after the big bang, which encircles the visible universe.

Earth is at the center of the visible universe. This does not mean that we are in a special location. If you look out from the top of the Empire State Building in New York City, the region you can see, out to the horizon, is circular and centered on the Empire State Building. Looking out from the top of a different building, you would see a different circular region—also centered on it. An observer in a distant galaxy would see a different visible region of the universe, centered on his galaxy instead of ours. Most of the galaxies visible are less than 5 billion light-years away, while most of the quasars are between 5 and 12 billion light-years away. Near Earth, many voids and walls of galaxies are visible. These also appear in the Map of the Universe shown on pages 133-136.

The Sloan Great Wall (highlighted by a detailed, semi-transparent line that points to its location in the map) is the largest structure we have found in the universe so far. Its length stretches 1.27 billion light-years, one-tenth the radius of the visible universe.

The visible universe is very large because gravity is such a weak force. In 1961, physicist Robert Dicke pointed out that it is no accident we live about one stellar main-sequence lifetime after the big bang—after some stars have died to make the carbon needed for life but before all the stars have burned out, making it too cold for life. If gravity were stronger, main-sequence lifetimes of stars would be shorter, we would live closer in time to the big bang, and the radius of the visible universe would be smaller. Since gravity is weak, we carbon-based life-forms, orbiting our main-sequence star, are treated to a truly grand view.

FYI We are now observing a scale a billion, billion billion times larger than Buzz Aldrin's footprint. And that footprint is a billion times larger than a hydrogen atom.

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Just think: After the end of inflation, the process that produced the big bang explosion 13.7 billion years ago, the matter we can see in the visible universe to day was still inside a region smaller than Buzz Aldrin's footprint.